14 Transistor amplifier

14.1 General information

The problem is a stiff DAE of index 1 consisting of 8 equations P. Rentrop has received it from K. Glashoff & H.J. Oberle and has documented it in [RRS89]. The formulation presented here has been taken from [HLR89]. The parallel-IVP-algorithm group of CWI contributed this problem to the test set.

The software part of the problem is in the file transamp.f available at [MM08].

14.2 Mathematical description of the problem

The problem is of the form

$$M\frac{\mathrm{d}y}{\mathrm{d}t} = f(t,y), \qquad y(0) = y_0, \quad y'(0) = y_0',$$

with

$$y \in \mathbb{R}^8, \quad 0 \le t \le 0.2.$$

The matrix M is of rank 5 and given by

$$M = \begin{pmatrix} -C_1 & C_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ C_1 & -C_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C_3 & C_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_3 & -C_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -C_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -C_5 & C_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & C_5 & -C_5 \end{pmatrix},$$

and the function f by

$$f(t,y) = \begin{pmatrix} -\frac{U_e(t)}{R_0} + \frac{y_1}{R_0} \\ -\frac{U_b}{R_2} + y_2(\frac{1}{R_1} + \frac{1}{R_2}) - (\alpha - 1)g(y_2 - y_3) \\ -g(y_2 - y_3) + \frac{y_3}{R_3} \\ -\frac{U_b}{R_4} + \frac{y_4}{R_4} + \alpha g(y_2 - y_3) \\ -\frac{U_b}{R_6} + y_5(\frac{1}{R_5} + \frac{1}{R_6}) - (\alpha - 1)g(y_5 - y_6) \\ -g(y_5 - y_6) + \frac{y_6}{R_7} \\ -\frac{U_b}{R_8} + \frac{y_7}{R_8} + \alpha g(y_5 - y_6) \\ \frac{y_8}{R_9} \end{pmatrix},$$

where g and U_e are auxiliary functions given by

$$g(x) = \beta(e^{\frac{x}{U_F}} - 1) \quad \text{and} \quad U_e(t) = 0.1\sin(200\pi t). \tag{II.14.1}$$

The values of the technical parameters are:

U_b	$=$	$6,$	R_0	$=$	$1000,$		
U_F	$=$	$0.026,$	R_k	$=$	9000	for	$k = 1, \ldots, 9,$
α	$=$	$0.99,$	C_k	$=$	$k \cdot 10^{-6}$	for	$k = 1, \ldots, 5.$
β	$=$	$10^{-6},$					

Consistent initial values at $t = 0$ are

$$y_0 = \begin{pmatrix} 0 \\ U_b/(\frac{R_2}{R_1}+1) \\ U_b/(\frac{R_2}{R_1}+1) \\ U_b \\ U_b/(\frac{R_6}{R_5}+1) \\ U_b/(\frac{R_6}{R_5}+1) \\ U_b \\ 0 \end{pmatrix}, \qquad y_0' = \begin{pmatrix} 51.338775 \\ 51.338775 \\ -U_b/((\frac{R_2}{R_1}+1)(C_2 \cdot R_3)) \\ -24.9757667 \\ -24.9757667 \\ -U_b/((\frac{R_6}{R_5}+1)(C_4 \cdot R_7)) \\ -10.00564453 \\ -10.00564453 \end{pmatrix}.$$

The first, fourth and seventh component of y_0' were determined numerically. All components of y are of index 1.

The definition of the function $g(x)$ in (II.14.1) may cause overflow if $\frac{x}{U_F}$ becomes too large. In the Fortran subroutines `feval` and `jeval` that define the function f and the partial derivatives of f with respect to y, respectively, we set `IERR=-1` if $\frac{x}{U_F} > 300$ to prevent this situation. See page IV-ix of the description of the software part of the test set for more details on `IERR`.

14.3 Origin of the problem

The problem originates from electrical circuit analysis. It is a model for the transistor amplifier. The diagram of the circuit is given in Figure II.14.1. Here U_e is the input signal and U_8 is the amplified

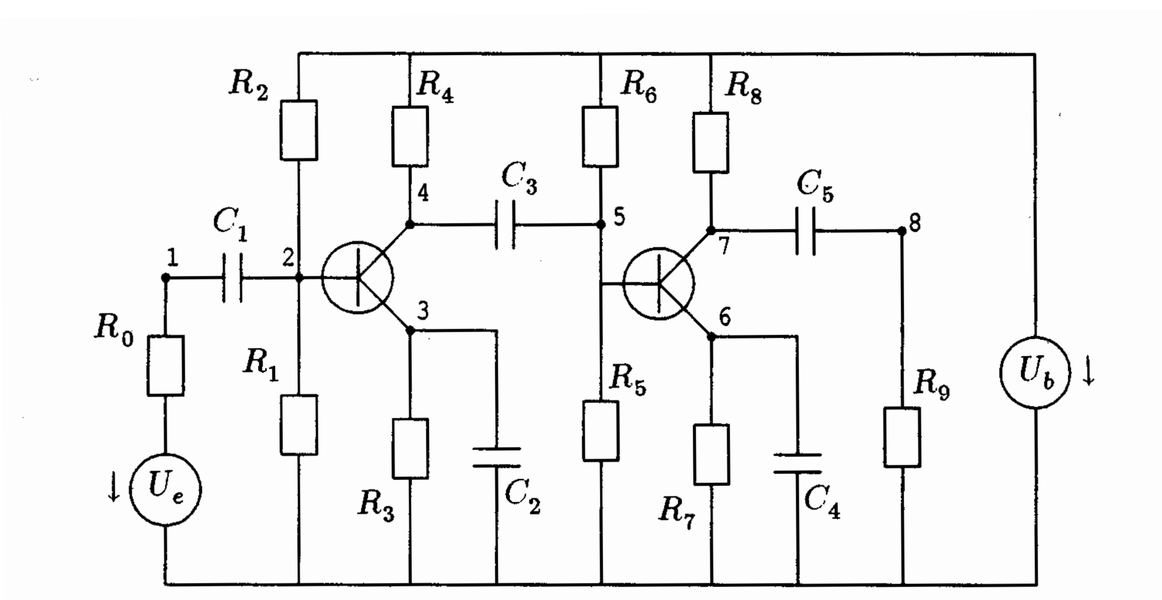

FIGURE II.14.1: *Circuit diagram of Transistor Amplifier (taken from [HLR89]).*

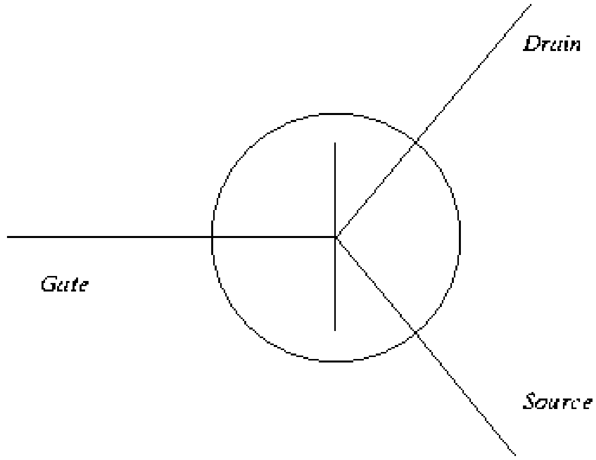

FIGURE II.14.2: *Schematic representation of a transistor.*

output voltage. The circuit contains two transistors of the form depicted in Figure II.14.2. As a simple model for the behavior of the transistors we assume that the currents through the gate, drain and source, which are denoted by I_G, I_D and I_S, respectively, are

$$\begin{aligned} I_G &= (1-\alpha)g(U_G - U_S), \\ I_D &= \alpha g(U_G - U_S), \\ I_S &= g(U_G - U_S), \end{aligned}$$

where U_G and U_S denote the voltage at the gate and source, respectively, and $\alpha = 0.99$. For the function g we take

$$g(U_i - U_j) = \beta(\mathrm{e}^{\frac{U_i - U_j}{U_F}} - 1),$$

where $\beta = 10^{-6}$ and $U_F = 0.026$.

To formulate the governing equations, Kirchoff's Current Law is used in each numbered node. This law states that the total sum of all currents entering a node must be zero. All currents passing through the circuit components can be expressed in terms of the unknown voltages $U_1, \ldots, U_8$. Consider for instance node 1. The current I_{C_1} passing through capacitor C_1 is given by

$$I_{C_1} = \frac{\mathrm{d}}{\mathrm{d}t}(C_1(U_2 - U_1)),$$

and the current I_{R_0} passing through the resistor R_0 by

$$I_{R_0} = \frac{U_e - U_1}{R_0}.$$

Here, the currents are directed towards node 1 if the current is positive. A similar derivation for the

Table II.14.1: *Failed runs.*

solver	m	reason
RADAU	$0,\ldots,8,30$	solver cannot handle `IERR=-1`.
RADAU5	$0,\ldots,8$	solver cannot handle `IERR=-1`.

other nodes gives the system:

$$\begin{array}{lll}
\text{node 1:} & \frac{\mathrm{d}}{\mathrm{d}t}(C_1(U_2-U_1))+\frac{U_e(t)}{R_0}-\frac{U_1}{R_0} & = 0,\\
\text{node 2:} & \frac{\mathrm{d}}{\mathrm{d}t}(C_1(U_1-U_2))+\frac{U_b}{R_2}-U_2(\frac{1}{R_1}+\frac{1}{R_2})+(\alpha-1)g(U_2-U_3) & = 0,\\
\text{node 3:} & -\frac{\mathrm{d}}{\mathrm{d}t}(C_2U_3)+g(U_2-U_3)-\frac{U_3}{R_3} & = 0,\\
\text{node 4:} & -\frac{\mathrm{d}}{\mathrm{d}t}(C_3(U_4-U_5))+\frac{U_b}{R_4}-\frac{U_4}{R_4}-\alpha g(U_2-U_3) & = 0,\\
\text{node 5:} & \frac{\mathrm{d}}{\mathrm{d}t}(C_3(U_4-U_5))+\frac{U_b}{R_6}-U_5(\frac{1}{R_5}+\frac{1}{R_6})+(\alpha-1)g(U_5-U_6) & = 0,\\
\text{node 6:} & -\frac{\mathrm{d}}{\mathrm{d}t}(C_4U_6)+g(U_5-U_6)-\frac{U_6}{R_7} & = 0,\\
\text{node 7:} & -\frac{\mathrm{d}}{\mathrm{d}t}(C_5(U_7-U_8))+\frac{U_b}{R_8}-\frac{U_7}{R_8}-\alpha g(U_5-U_6) & = 0.\\
\text{node 8:} & -\frac{\mathrm{d}}{\mathrm{d}t}(C_5(U_7-U_8))+\frac{U_8}{R_9} & = 0,
\end{array}$$

The input signal $U_e(t)$ is

$$U_e(t) = 0.1\sin(200\pi t).$$

To arrive at the mathematical formulation of the preceding subsection, one just has to identify U_i with y_i.

From the plot of output signal $U_8 = y(8)$ in Figure II.14.2 we see that the amplitude of the input signal U_e is indeed amplified.

14.4 Numerical solution of the problem

Tables II.14.2–II.14.3 and Figures II.14.3–II.14.4 present the reference solution at the end of the integration interval, the run characteristics, the behavior of the solution over the integration interval and the work-precision diagrams, respectively. The reference solution was computed on the Cray C90, using PSIDE with Cray double precision and atol = rtol = 10^{-14}. For the work-precision diagrams, we used: rtol = $10^{-(4+m/8)}$, $m = 0,1,\ldots,40$; atol = rtol; h0 = $10^{-2}\cdot$ rtol for BIMD, GAMD, MEBDFDAE, MEBDFI, RADAU and RADAU5.

The failed runs are in Table II.14.1; listed are the name of the solver that failed, for which values of m this happened, and the reason for failing.

References

[HLR89] E. Hairer, C. Lubich, and M. Roche. *The Numerical Solution of Differential-Algebraic Systems by Runge–Kutta Methods.* Lecture Notes in Mathematics 1409. Springer-Verlag, 1989.

TABLE II.14.2: *Reference solution at the end of the integration interval.*

y_1	$-0.5562145012262709 \cdot 10^{-2}$	y_5	$0.2704617865010554 \cdot 10$
y_2	$0.3006522471903042 \cdot 10$	y_6	$0.2761837778393145 \cdot 10$
y_3	$0.2849958788608128 \cdot 10$	y_7	$0.4770927631616772 \cdot 10$
y_4	$0.2926422536206241 \cdot 10$	y_8	$0.1236995868091548 \cdot 10$

TABLE II.14.3: *Run characteristics.*

solver	rtol	atol	h0	mescd	scd	steps	accept	#f	#Jac	#LU	CPU
BIMD	10^{-4}	10^{-4}	10^{-6}	5.85	5.63	466	408	8423	408	466	0.0322
	10^{-7}	10^{-7}	10^{-9}	8.62	8.34	618	575	19632	575	618	0.0752
DDASSL	10^{-4}	10^{-4}		4.60	3.08	9759	6026	18381	7359		0.1113
	10^{-7}	10^{-7}		7.24	5.49	40810	23859	77402	33678		0.4743
GAMD	10^{-4}	10^{-4}	10^{-6}	6.30	5.83	373	276	17204	276	373	0.0517
	10^{-7}	10^{-7}	10^{-9}	8.58	7.37	374	325	34320	326	374	0.1064
MEBDFI	10^{-4}	10^{-4}	10^{-6}	5.06	4.80	1580	1486	5949	256	256	0.0303
	10^{-7}	10^{-7}	10^{-9}	7.25	6.99	3628	3513	13324	419	419	0.0703
PSIDE-1	10^{-4}	10^{-4}		5.02	4.76	516	362	9742	253	2008	0.0351
	10^{-7}	10^{-7}		7.50	7.23	835	653	21914	419	2724	0.0732
RADAU	10^{-7}	10^{-7}	10^{-9}	7.11	6.83	1775	1551	17582	1541	1775	0.0517

[MM08] F. Mazzia and C. Magherini. *Test Set for Initial Value Problem Solvers, release 2.4.* Department of Mathematics, University of Bari and INdAM, Research Unit of Bari, February 2008. Available at http://www.dm.uniba.it/~testset.

[RRS89] P. Rentrop, M. Roche, and G. Steinebach. The application of Rosenbrock-Wanner type methods with stepsize control in differential-algebraic equations. *Numer. Math.*, 55:545–563, 1989.

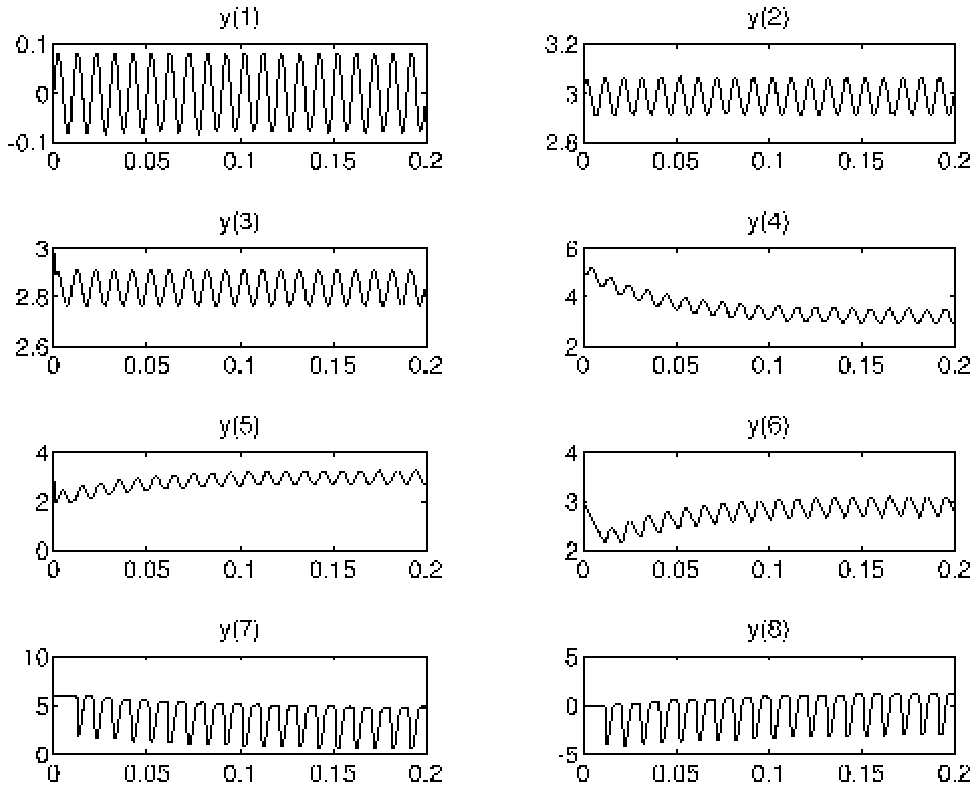

FIGURE II.14.3: *Behavior of the solution over the integration interval.*

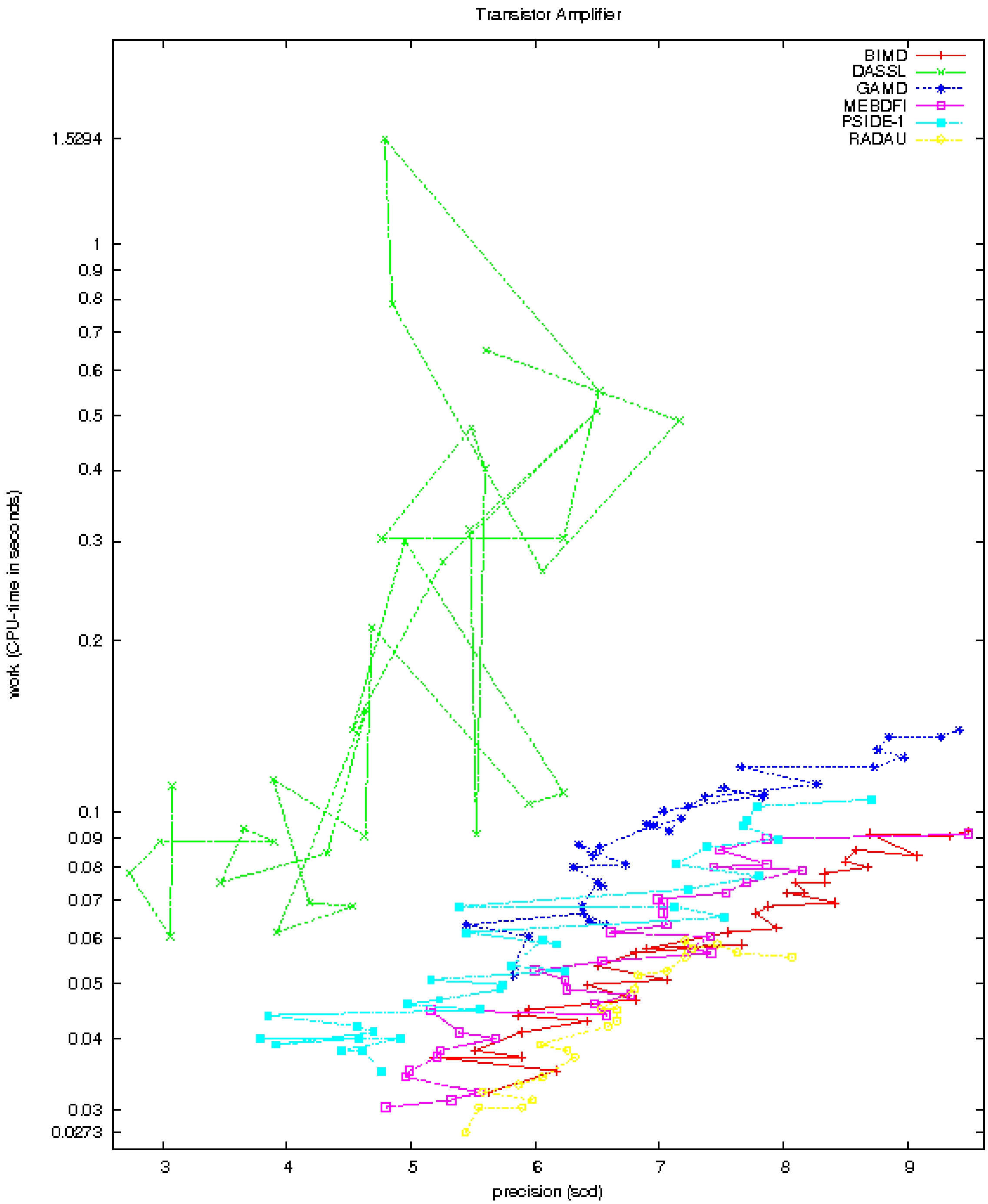

FIGURE II.14.4: *Work-precision diagram (scd versus CPU-time).*

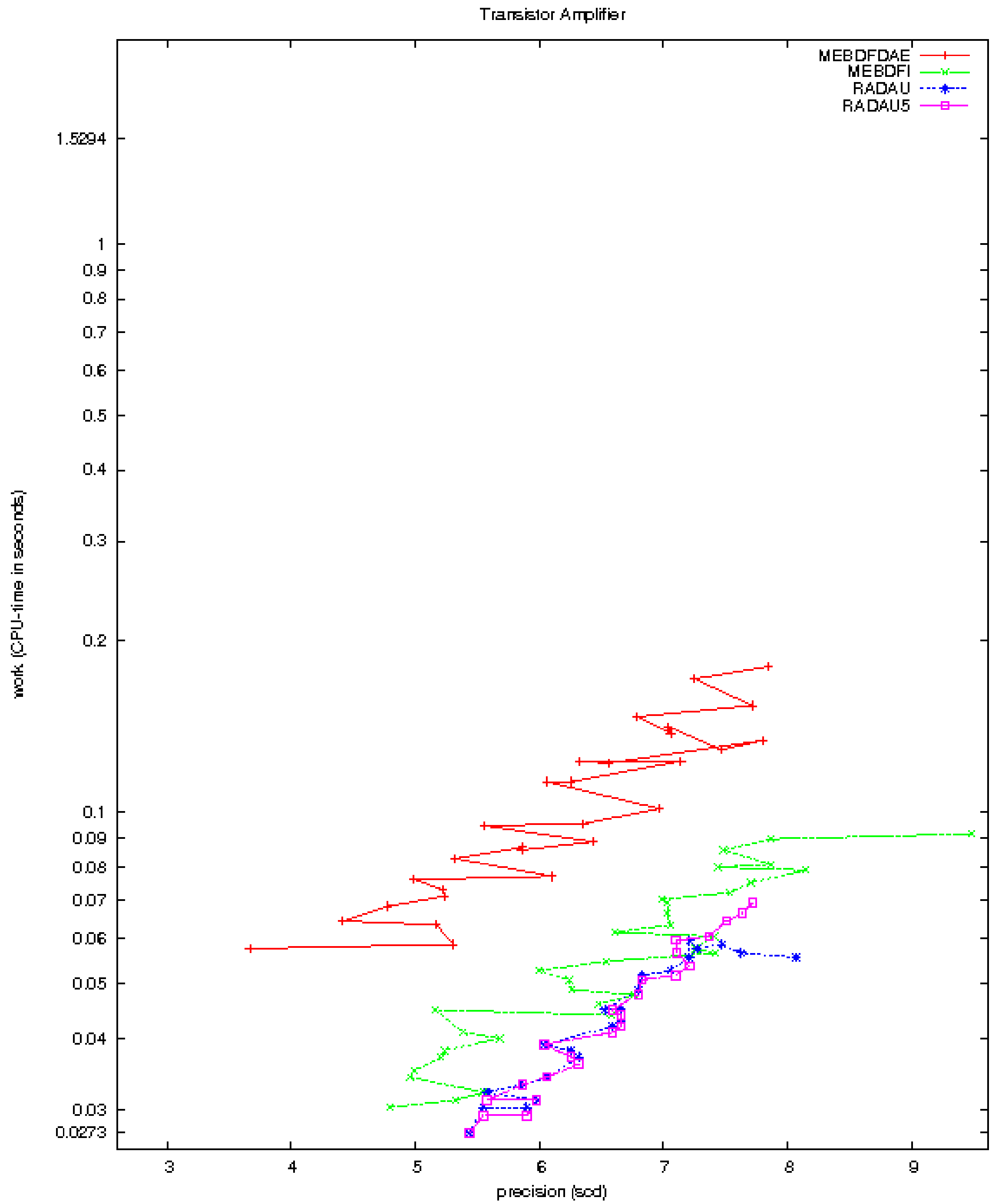

FIGURE II.14.5: *Work-precision diagram (scd versus CPU-time).*

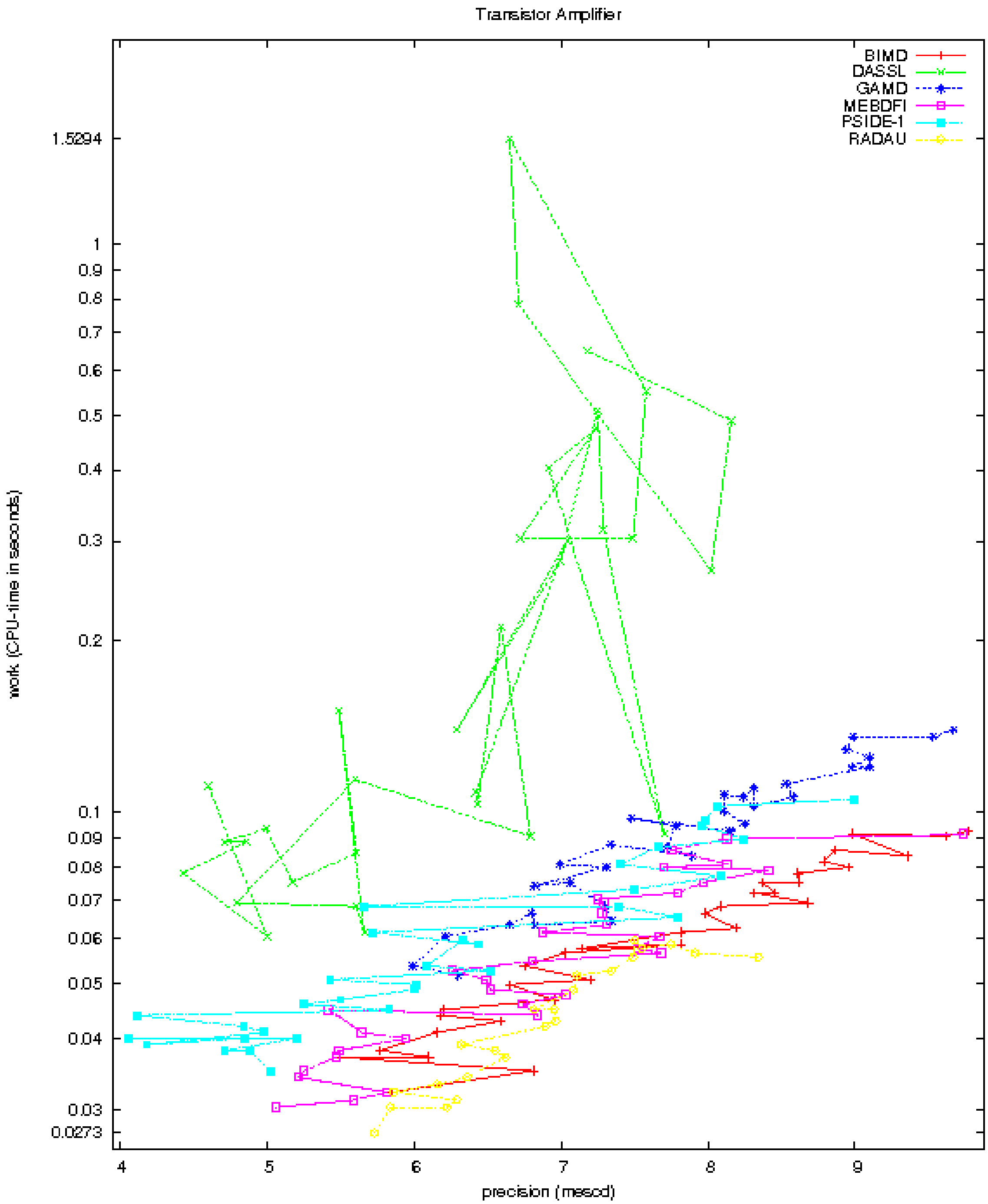

FIGURE II.14.6: *Work-precision diagram (mescd versus CPU-time).*

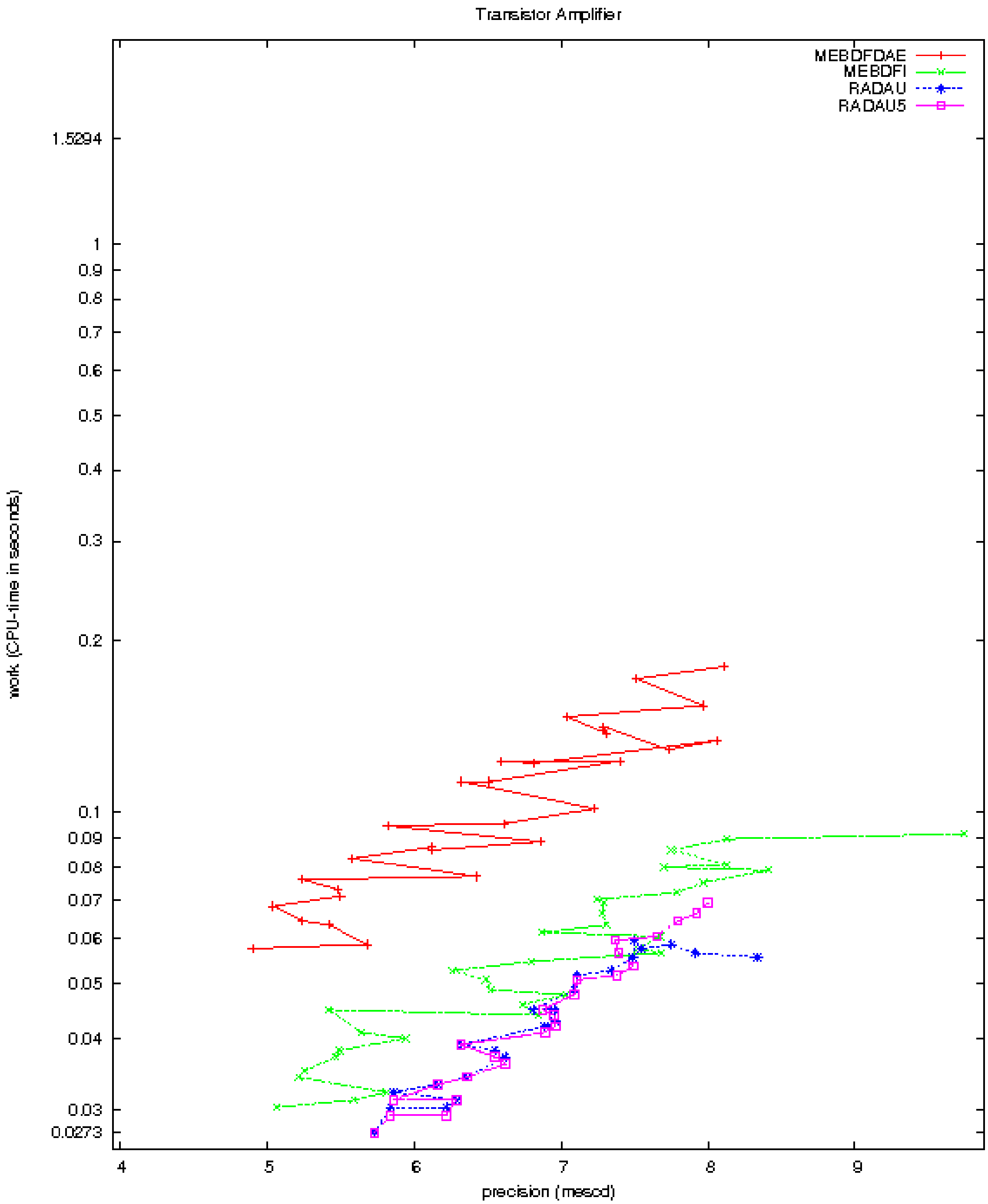

FIGURE II.14.7: *Work-precision diagram (mescd versus CPU-time).*