3 Ring modulator

3.1 General information

The type of the problem depends on the parameter C_s. If $C_s \neq 0$, then it is a stiff system of 15 non-linear ordinary differential equations. For $C_s = 0$ we have a DAE of index 2, consisting of 11 differential equations and 4 algebraic equations. The numerical results presented here refer to $C_s = 2 \cdot 10^{-12}$. The problem has been taken from [KRS92], where the approach of Horneber [Hor76] is followed. The parallel-IVP-algorithm group of CWI contributed this problem to the test set.

The software part of the problem is in the file ringmod.f available at [MM08].

3.2 Mathematical description of the problem

For the ODE case, the problem is of the form

$$\frac{\mathrm{d}y}{\mathrm{d}t} = f(t,y), \quad y(0) = y_0,$$

with

$$y \in \mathrm{I\!R}^{15}, \quad 0 \leq t \leq 10^{-3}.$$

The function f is defined by

$$f(t,y) = \begin{pmatrix} C^{-1}(y_8 - 0.5y_{10} + 0.5y_{11} + y_{14} - R^{-1}y_1) \\ C^{-1}(y_9 - 0.5y_{12} + 0.5y_{13} + y_{15} - R^{-1}y_2) \\ C_s^{-1}(y_{10} - q(U_{D1}) + q(U_{D4})) \\ C_s^{-1}(-y_{11} + q(U_{D2}) - q(U_{D3})) \\ C_s^{-1}(y_{12} + q(U_{D1}) - q(U_{D3})) \\ C_s^{-1}(-y_{13} - q(U_{D2}) + q(U_{D4})) \\ C_p^{-1}(-R_p^{-1}y_7 + q(U_{D1}) + q(U_{D2}) - q(U_{D3}) - q(U_{D4})) \\ -L_h^{-1}y_1 \\ -L_h^{-1}y_2 \\ L_{s2}^{-1}(0.5y_1 - y_3 - R_{g2}y_{10}) \\ L_{s3}^{-1}(-0.5y_1 + y_4 - R_{g3}y_{11}) \\ L_{s2}^{-1}(0.5y_2 - y_5 - R_{g2}y_{12}) \\ L_{s3}^{-1}(-0.5y_2 + y_6 - R_{g3}y_{13}) \\ L_{s1}^{-1}(-y_1 + U_{in1}(t) - (R_i + R_{g1})y_{14}) \\ L_{s1}^{-1}(-y_2 - (R_c + R_{g1})y_{15}) \end{pmatrix}. \tag{II.3.1}$$

The auxiliary functions $U_{D1}, U_{D2}, U_{D3}, U_{D4}, q, U_{in1}$ and U_{in2} are given by

$$\begin{aligned} U_{D1} &= y_3 - y_5 - y_7 - U_{in2}(t), \\ U_{D2} &= -y_4 + y_6 - y_7 - U_{in2}(t), \\ U_{D3} &= y_4 + y_5 + y_7 + U_{in2}(t), \\ U_{D4} &= -y_3 - y_6 + y_7 + U_{in2}(t), \\ q(U) &= \gamma(e^{\delta U} - 1), \\ U_{in1}(t) &= 0.5\sin(2000\pi t), \\ U_{in2}(t) &= 2\sin(20000\pi t). \end{aligned} \tag{II.3.2}$$

The values of the parameters are:

C	=	$1.6 \cdot 10^{-8}$	R	=	25000
C_s	=	$2 \cdot 10^{-12}$	R_p	=	50
C_p	=	10^{-8}	R_{g1}	=	36.3
L_h	=	4.45	R_{g2}	=	17.3
L_{s1}	=	0.002	R_{g3}	=	17.3
L_{s2}	=	$5 \cdot 10^{-4}$	R_i	=	50
L_{s3}	=	$5 \cdot 10^{-4}$	R_c	=	600
γ	=	$40.67286402 \cdot 10^{-9}$	δ	=	17.7493332

The initial vector y_0 is given by

$$y_0 = (0,0,0,0,0,0,0,0,0,0,0,0,0,0,0)^{\mathrm{T}}.$$

The definition of the function $q(U)$ in (II.3.2) may cause overflow if δU becomes too large. In the Fortran subroutine that defines f, we set `IERR=-1` if $\delta U > 300$ to prevent this situation. See page *IV-ix* of the description of the software part of the test set for more details on `IERR`.

3.3 Origin of the problem

The problem originates from electrical circuit analysis. It describes the behavior of the ring modulator, of which the circuit diagram is given in Figure II.3.1. Given a low-frequency signal U_{in1} and a high-frequency signal U_{in2}, the ring modulator produces a mixed signal in U_2.

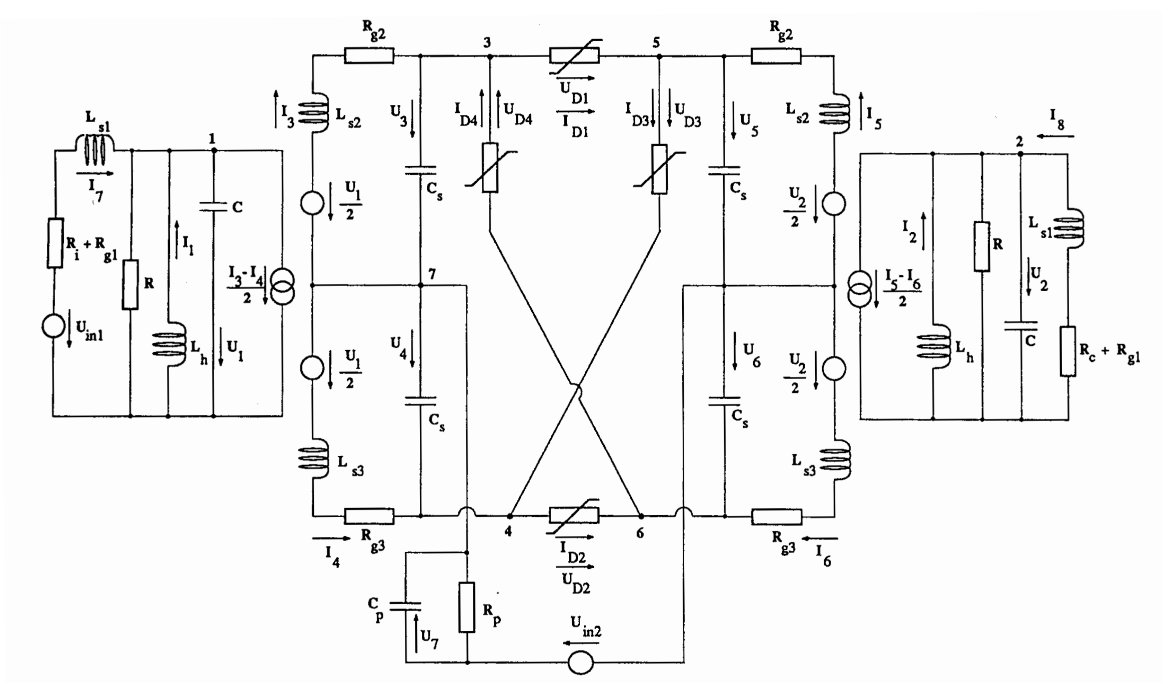

FIGURE II.3.1: *Circuit diagram for Ring Modulator (taken from [KRS92]).*

Every capacitor in the diagram leads to a differential equation:

$$C\dot{U} = I.$$

Applying Kirchhoff's Current Law yields the following differential equations:

$$\begin{aligned}
C\dot{U}_1 &= I_1 - 0.5I_3 + 0.5I_4 + I_7 - R^{-1}U_1, \\
C\dot{U}_2 &= I_2 - 0.5I_5 + 0.5I_6 + I_8 - R^{-1}U_2, \\
C_s\dot{U}_3 &= I_3 - q(U_{D1}) + q(U_{D4}), \\
C_s\dot{U}_4 &= -I_4 + q(U_{D2}) - q(U_{D3}), \\
C_s\dot{U}_5 &= I_5 + q(U_{D1}) - q(U_{D3}), \\
C_s\dot{U}_6 &= -I_6 - q(U_{D2}) + q(U_{D4}), \\
C_p\dot{U}_7 &= -R_p^{-1}U_7 + q(U_{D1}) + q(U_{D2}) - q(U_{D3}) - q(U_{D4}),
\end{aligned}$$

where U_{D1}, U_{D2}, U_{D3} and U_{D4} stand for:

$$\begin{aligned}
U_{D1} &= U_3 - U_5 - U_7 - U_{in2}, \\
U_{D2} &= -U_4 + U_6 - U_7 - U_{in2}, \\
U_{D3} &= U_4 + U_5 + U_7 + U_{in2}, \\
U_{D4} &= -U_3 - U_6 + U_7 + U_{in2}.
\end{aligned}$$

The diode function q is given by

$$q(U) = \gamma(e^{\delta U} - 1),$$

where γ and δ are fixed constants.

Every inductor leads to a differential equation as well:

$$L\dot{I} = U.$$

Applying Kirchoff's Voltage Law to closed loops that contains an inductor, results in another 8 differential equations:

$$\begin{aligned}
L_h\dot{I}_1 &= -U_1, \\
L_h\dot{I}_2 &= -U_2, \\
L_{s2}\dot{I}_3 &= 0.5U_1 - U_3 - R_{g2}I_3, \\
L_{s3}\dot{I}_4 &= -0.5U_1 + U_4 - R_{g3}I_4, \\
L_{s2}\dot{I}_5 &= 0.5U_2 - U_5 - R_{g2}I_5, \\
L_{s3}\dot{I}_6 &= -0.5U_2 + U_6 - R_{g3}I_6, \\
L_{s1}\dot{I}_7 &= -U_1 + U_{in1} - (R_i + R_{g1})I_7, \\
L_{s1}\dot{I}_8 &= -U_2 - (R_c + R_{g1})I_8.
\end{aligned}$$

Initially, all voltages and currents are zero.

Identifying the voltages with $y_1, \ldots, y_7$ and the currents with $y_8, \ldots, y_{15}$, we obtain the 15 differential equations (II.3.1). From the plot of $y_2 = U_2$ in Figure II.3.2 we see how the low and high frequency input signals are mixed by the ring modulator.

3.4 Numerical solution of the problem

Tables II.3.2–II.3.3 and Figures II.3.2–II.3.7 present the reference solution at the end of the integration interval, the run characteristics, the behavior of the solution over the integration interval and the work-precision diagrams, respectively. The reference solution was computed using PSIDE with `atol = rtol =` 10^{-13}. For the work-precision diagrams, we used: rtol $= 10^{-(4+m/4)}$, $m = 0, 1, \ldots, 32$; atol = rtol; h0 $= 10^{-2} \cdot$ rtol for BIMD, GAMD, MEBDFDAE, MEBDFI, RADAU and RADAU5. The failed runs are in Table II.3.1; listed are the name of the solver that failed, for which values of m this happened, and the reason for failing.

TABLE II.3.1: *Failed runs.*

solver	m	reason
RADAU	$0, 1, \ldots, 26$	solver cannot handle `IERR=-1`.
RADAU5	$0, 1, \ldots, 9$	solver cannot handle `IERR=-1`.
VODE	0	solver cannot handle `IERR=-1`.
VODE	2	error test failed repeatedly.

TABLE II.3.2: *Reference solution at the end of the integration interval.*

y_1	$-0.2339057358486745 \cdot 10^{-1}$	y_9	$-0.2840029933642329 \cdot 10^{-7}$
y_2	$-0.7367485485540825 \cdot 10^{-2}$	y_{10}	$0.7267198267264553 \cdot 10^{-3}$
y_3	0.2582956709291169	y_{11}	$0.7929487196960840 \cdot 10^{-3}$
y_4	-0.4064465721283450	y_{12}	$-0.7255283495698965 \cdot 10^{-3}$
y_5	-0.4039455665149794	y_{13}	$-0.7941401968526521 \cdot 10^{-3}$
y_6	0.2607966765422943	y_{14}	$0.7088495416976114 \cdot 10^{-4}$
y_7	0.1106761861269975	y_{15}	$0.2390059075236570 \cdot 10^{-4}$
y_8	$0.2939904342435596 \cdot 10^{-6}$		

TABLE II.3.3: *Run characteristics.*

solver	rtol	atol	h0	mescd	scd	steps	accept	#f	#Jac	#LU	CPU
BIMD	10^{-4}	10^{-4}	10^{-6}	2.89	2.20	19415	19089	455877	17614	19127	3.0793
	10^{-7}	10^{-7}	10^{-9}	7.08	6.28	26590	25880	824318	25865	26585	5.5886
DDASSL	10^{-4}	10^{-4}		1.18	0.49	88627	86091	116778	3538		1.4230
	10^{-7}	10^{-7}		3.22	2.53	252827	249239	318196	7777		4.0123
GAMD	10^{-4}	10^{-4}	10^{-6}	2.34	1.65	12420	11264	474866	11264	12420	2.7572
	10^{-7}	10^{-7}	10^{-9}	6.11	5.42	18798	16913	1049423	16909	18793	6.0502
MEBDFI	10^{-4}	10^{-4}	10^{-6}	2.54	1.85	61426	61208	201899	5374	5374	1.6416
	10^{-7}	10^{-7}	10^{-9}	5.28	4.59	148609	148298	483689	12471	12471	3.9831
PSIDE-1	10^{-4}	10^{-4}		1.29	0.60	9791	8241	267721	6834	38184	1.6709
	10^{-7}	10^{-7}		5.21	4.53	55345	45636	886724	3984	111508	5.4656
RADAU5	10^{-7}	10^{-7}	10^{-9}	4.49	3.80	102515	93113	545282	12316	54746	3.7742
VODE	10^{-7}	10^{-7}		2.84	2.15	217383	207569	261396	3605	22598	2.4019

References

[Hor76] E.H. Horneber. *Analyse nichtlinearer RLCÜ-Netzwerke mit Hilfe der gemischten Potentialfunktion mit einer systematischen Darstellung der Analyse nichtlinearer dynamischer Netzwerke*. PhD thesis, Universität Kaiserslautern, 1976.

[KRS92] W. Kampowski, P. Rentrop, and W. Schmidt. Classification and numerical simulation of electric circuits. *Surveys on Mathematics for Industry*, 2(1):23–65, 1992.

[MM08] F. Mazzia and C. Magherini. *Test Set for Initial Value Problem Solvers, release 2.4*. Department of Mathematics, University of Bari and INdAM, Research Unit of Bari, February 2008. Available at http://www.dm.uniba.it/~testset.

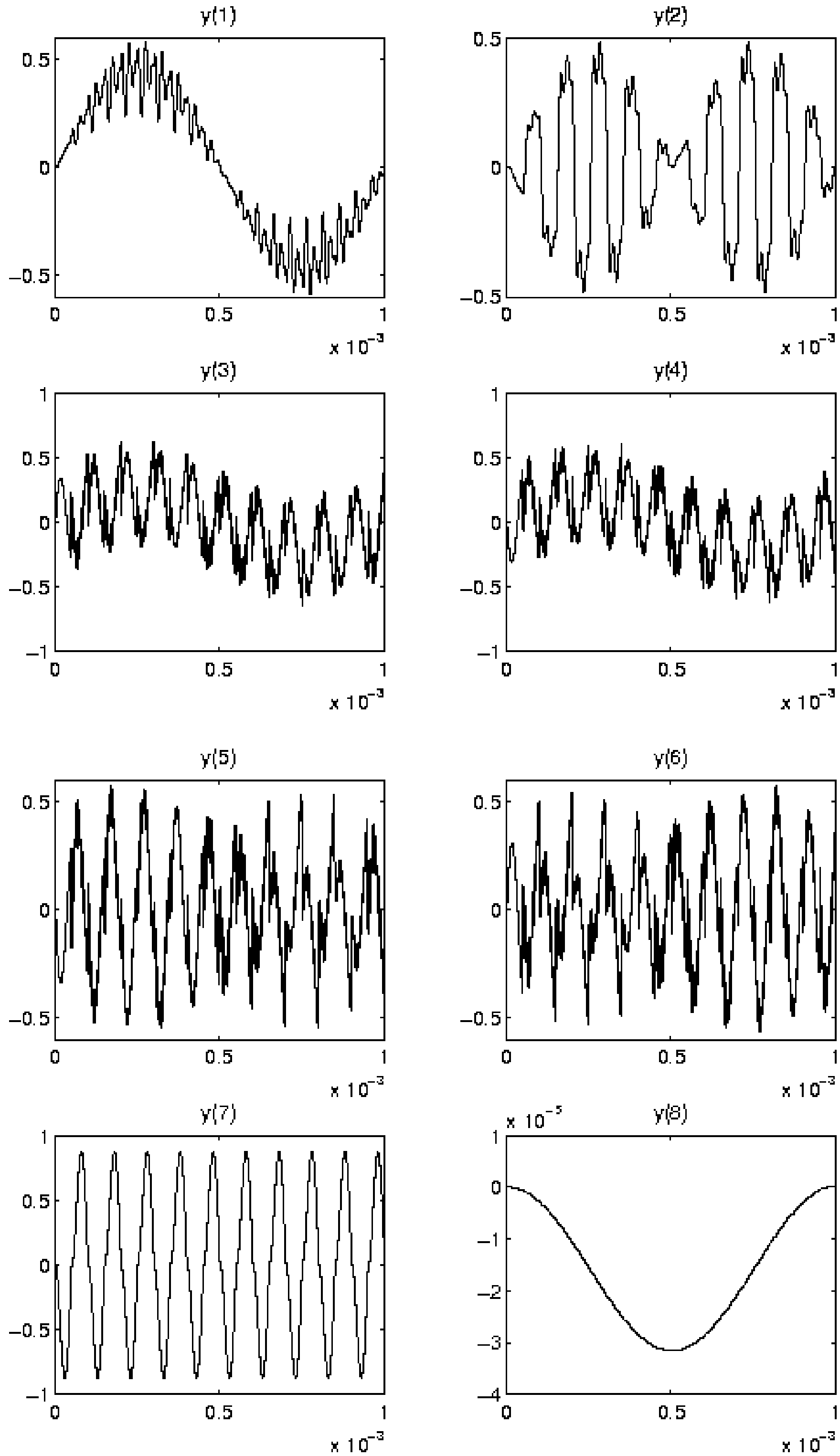

FIGURE II.3.2: *Behavior of the first eight solution components solution over the integration interval.*

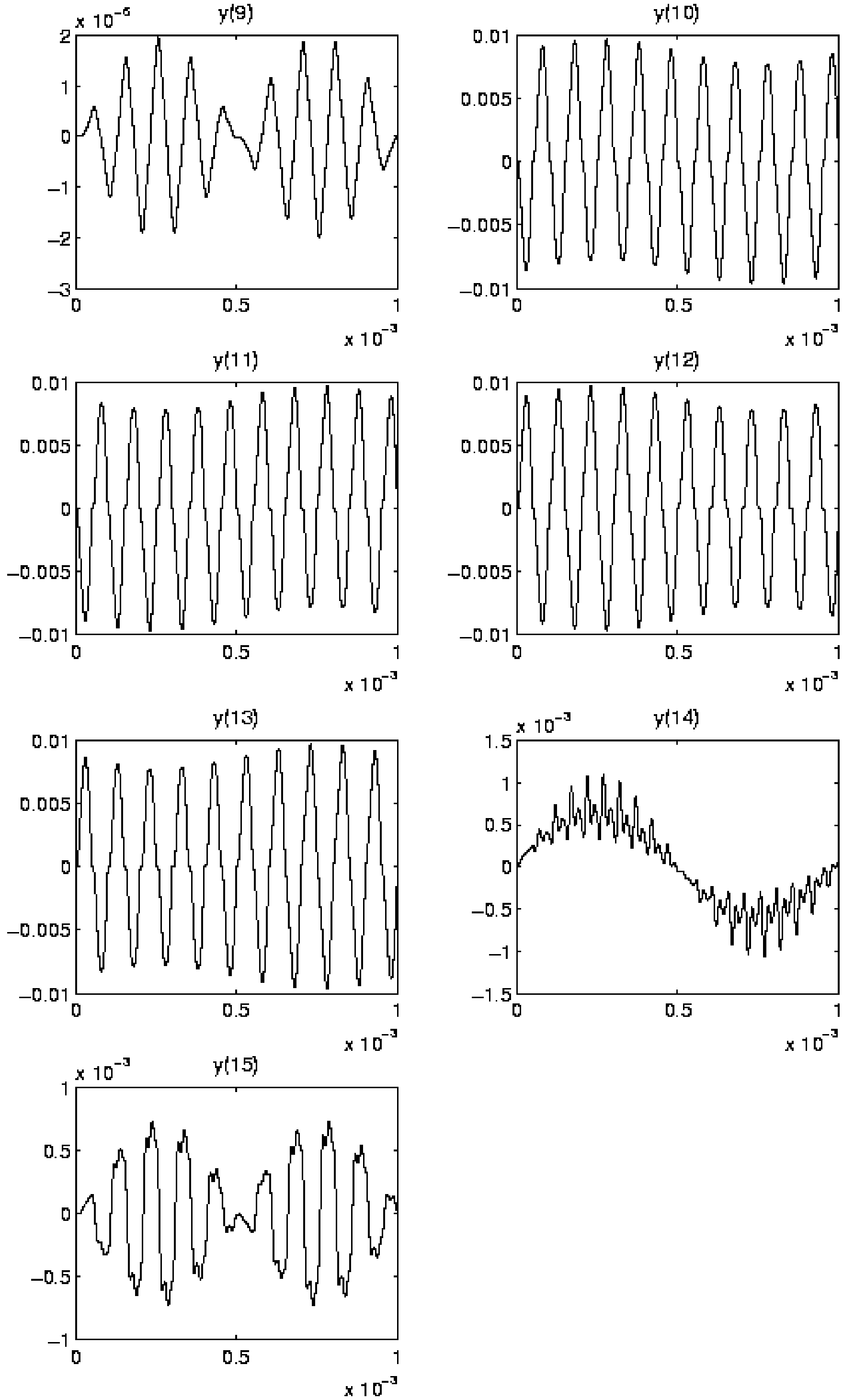

FIGURE II.3.3: *Behavior of the last seven solution components solution over the integration interval.*

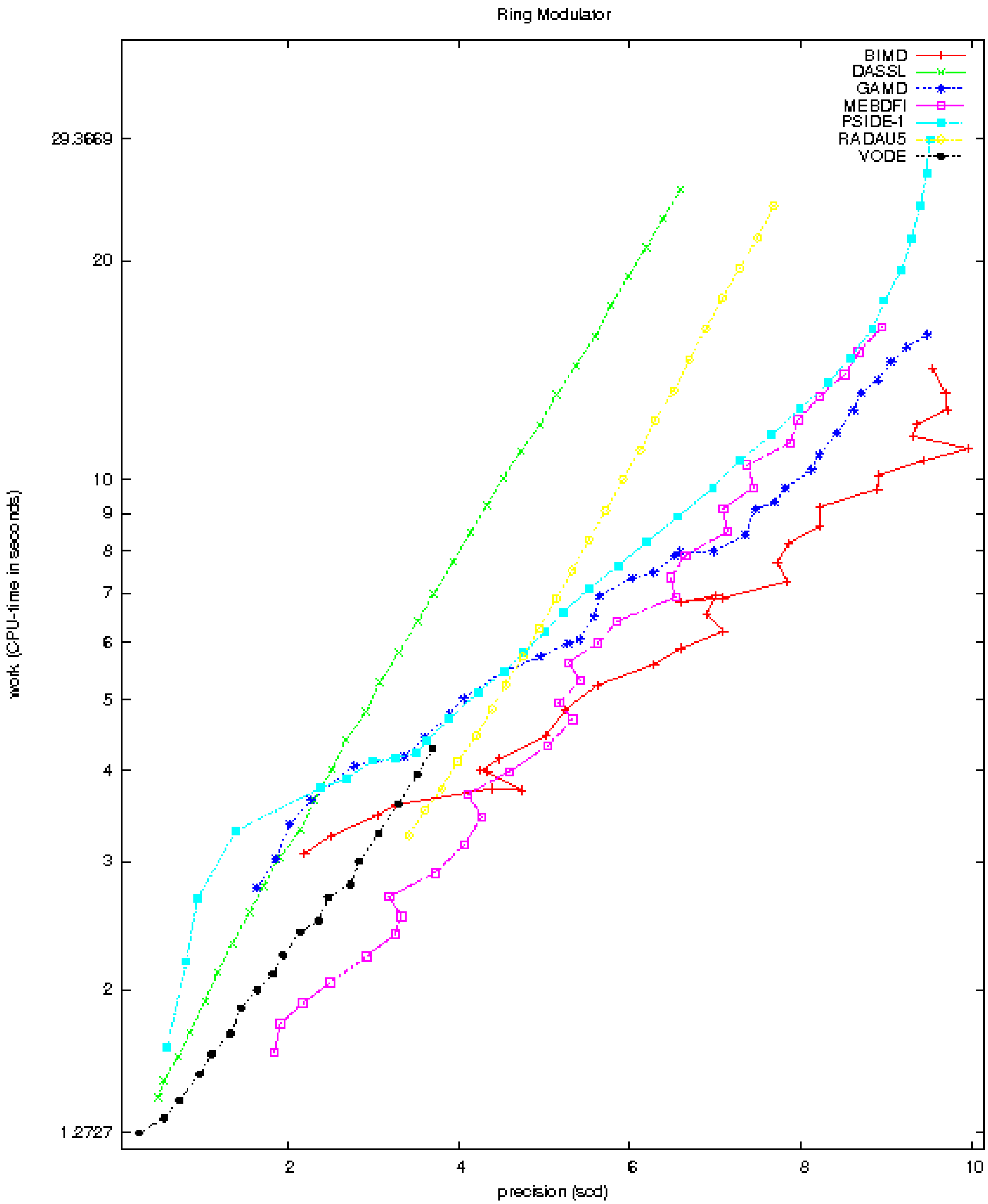

FIGURE II.3.4: *Work-precision diagram (scd versus CPU-time).*

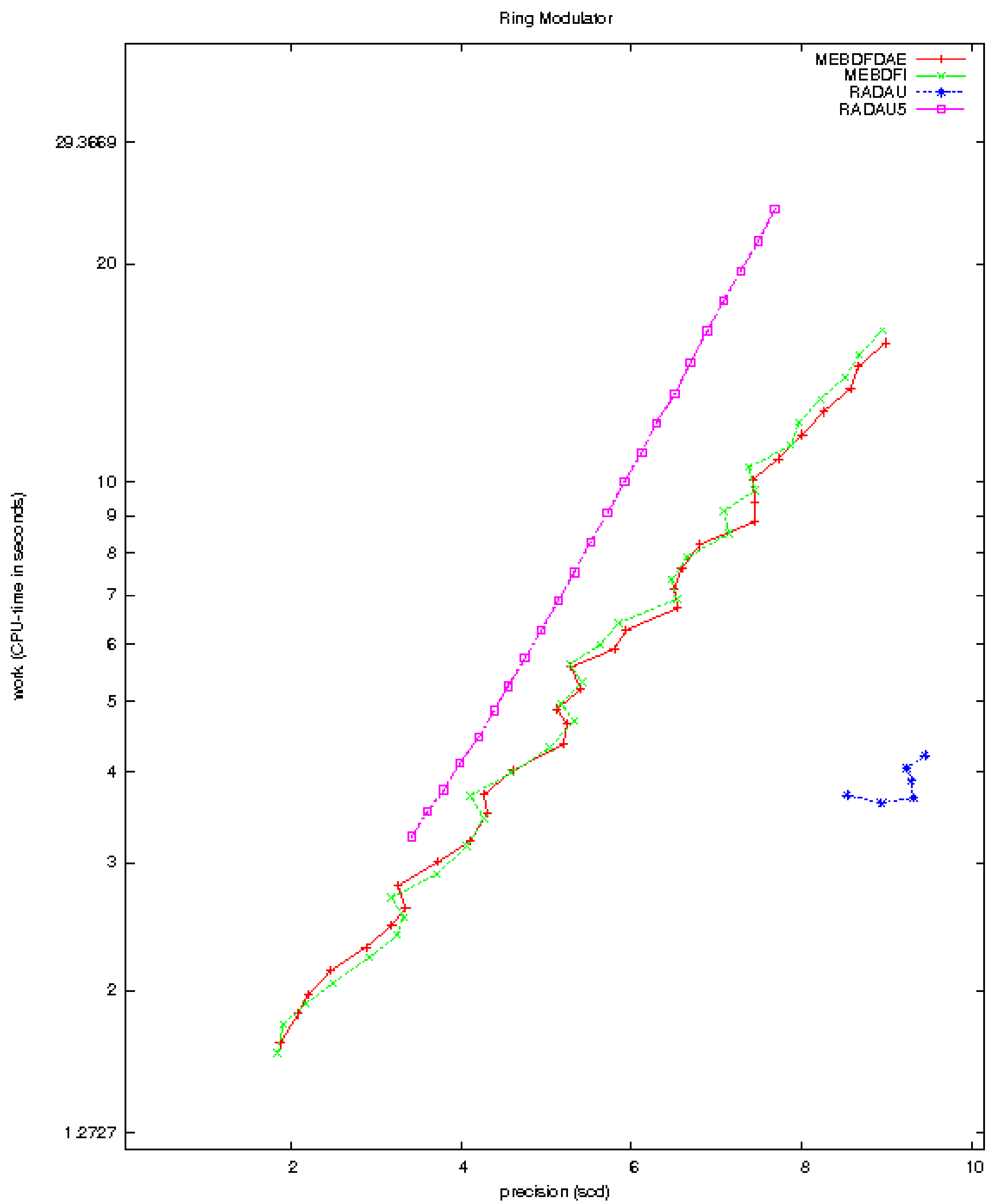

FIGURE II.3.5: *Work-precision diagram (scd versus CPU-time).*

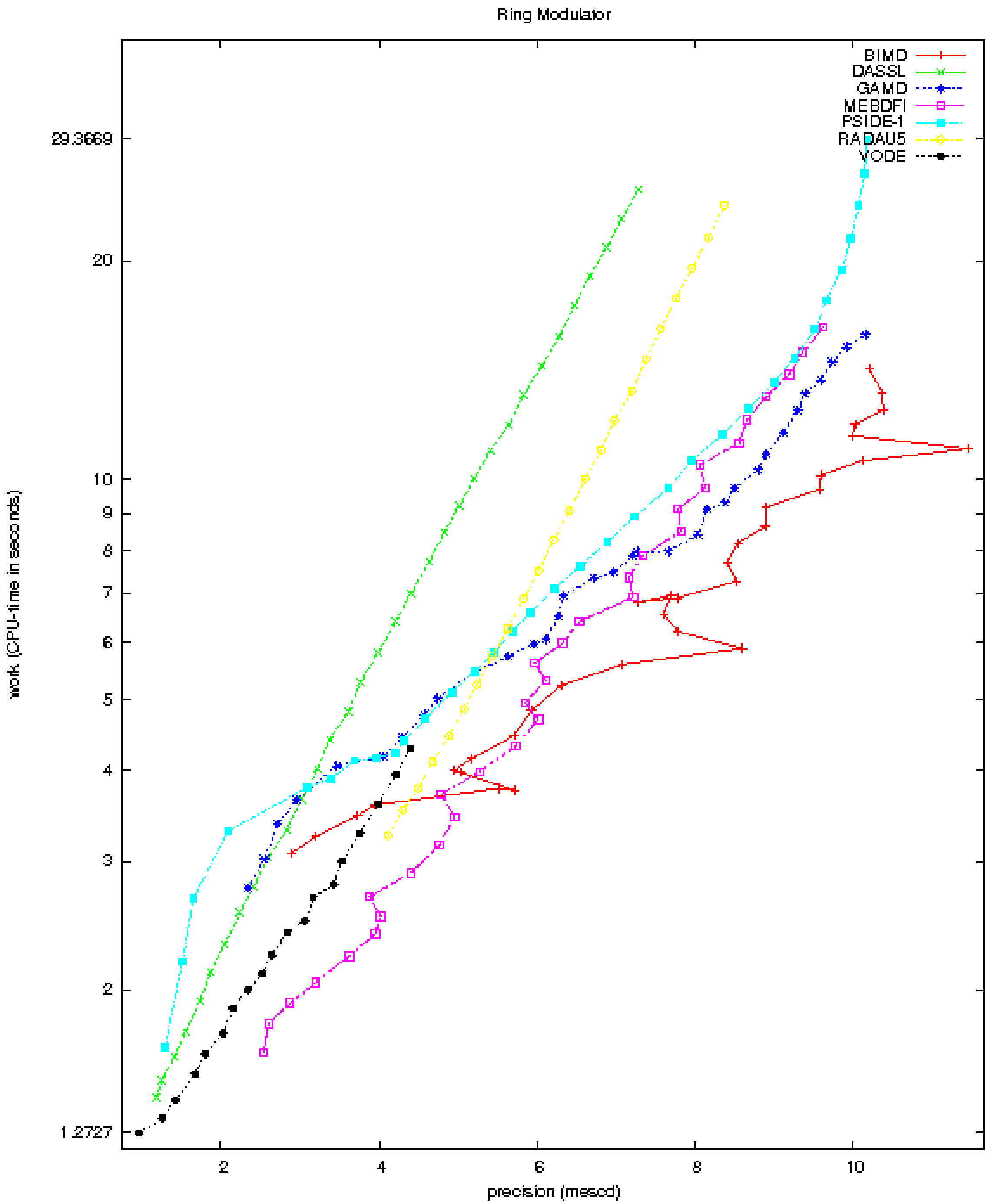

FIGURE II.3.6: *Work-precision diagram (mescd versus CPU-time).*

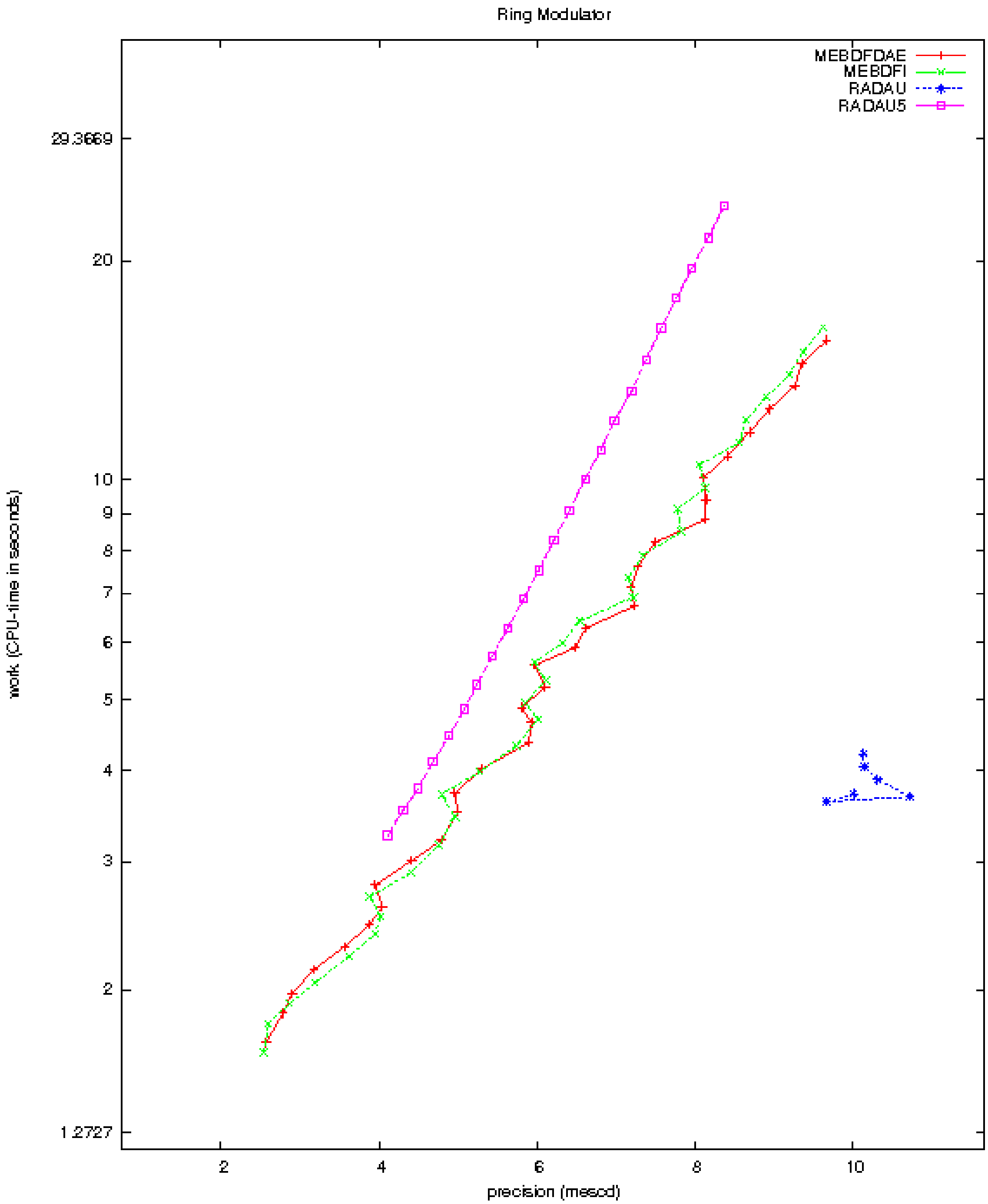

FIGURE II.3.7: *Work-precision diagram (mescd versus CPU-time).*