15 Charge pump

15.1 General information

The problem is a stiff DAE of index 2, consisting of 3 differential and 6 algebraic equations. It has been contributed by Michael Günther, Georg Denk and Uwe Feldmann [GDF95].

The software part of the problem is in the file pump.f available at [MM08].

15.2 Mathematical description

The problem is of the form

$$M\frac{\mathrm{d}y}{\mathrm{d}t} = f(t, y(t)), \qquad y(0) = y_0, \quad y'(0) = y_0',$$

with

$$y \in \mathbb{R}^9, \qquad 0 \le t \le 1.2 \cdot 10^{-6}.$$

The 9×9 matrix M is the zero matrix except for the the minor $M_{1..3,1..5}$, that is given by

$$M_{1..3,1..5} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}.$$

The function f is defined by

$$f(t,y) = \begin{pmatrix} -y_9 \\ 0 \\ 0 \\ -y_6 + V_{in}(t) \\ y_1 - Q_G(v) \\ y_2 - C_S \cdot y_7 \\ y_3 - Q_S(v) \\ y_4 - C_D \cdot y_8 \\ y_5 - Q_D(v) \end{pmatrix},$$

with $v := (v_1, v_2, v_3) = (y_6, y_6 - y_7, y_6 - y_8)$, $C_D = 0.4 \cdot 10^{-12}$ and $C_S = 1.6 \cdot 10^{-12}$. The functions Q_G, Q_S and Q_D are given by:

1. If $v_1 \le V_{FB} := U_{T0} - \gamma\sqrt{\Phi} - \Phi$, then

$$\begin{aligned} Q_G(v) &= C_{ox}(v_1 - V_{FB}), \\ Q_S(v) &= Q_D(v) = 0, \end{aligned}$$

   with $C_{ox} = 4 \cdot 10^{-12}$, $U_{T0} = 0.2$, $\gamma = 0.035$ and $\Phi = 1.01$.

2. If $v_1 > V_{FB}$ and $v_2 \le U_{TE} := U_{T0} + \gamma(\sqrt{\Phi - U_{BS}} - \sqrt{\Phi})$, then

$$\begin{aligned} Q_G(v) &= C_{ox}\gamma\left(\sqrt{(\gamma/2)^2 + v_1 - V_{FB}} - \gamma/2\right), \\ Q_S(v) &= Q_D(v) = 0. \end{aligned}$$

3. If $v_1 > V_{FB}$ and $v_2 > U_{TE}$, then

$$\begin{aligned} Q_G(v) &= C_{ox}\left(\frac{2}{3}(U_{GDT} + U_{GST} - \frac{U_{GDT}U_{GST}}{U_{GDT} + U_{GST}}) + \gamma\sqrt{\Phi - U_{BS}}\right), \\ Q_S(v) &= Q_D(v) = -\frac{1}{2}\left(Q_G - C_{ox}\gamma\sqrt{\Phi - U_{BS}}\right). \end{aligned}$$

Here, U_{BS}, U_{GST} and U_{GDT} are given by

$$\begin{aligned} U_{BS} &= v_2 - v_1, \\ U_{GST} &= v_2 - U_{TE}, \\ U_{GDT} &= \begin{cases} v_3 - U_{TE} & \text{for} \quad v_3 > U_{TE}, \\ 0 & \text{for} \quad v_3 \le U_{TE}. \end{cases} \end{aligned}$$

The function $V_{in}(t)$ is defined using $\tau = (10^9 \cdot t) \bmod 120$ by

$$V_{in}(t) = \begin{cases} 0 & \text{if} \quad \tau < 50, \\ 20(\tau - 50) & \text{if} \quad 50 \le \tau < 60, \\ 20 & \text{if} \quad 60 \le \tau < 110, \\ 20(120 - \tau) & \text{if} \quad \tau \ge 110. \end{cases}$$

This means that the function f has discontinuities in its derivative at $\tau = 50, 60, 90, 110, 120$.

Consistent initial values are

$$y_0 = (Q_G(0,0,0), 0, Q_S(0,0,0), 0, Q_D(0,0,0), 0, 0, 0, 0)^T \quad \text{and} \quad y_0' = (0,0,0,0,0,0,0,0,0)^T.$$

The index of the first eight variables is 1, whereas the index of y_9 is 2.

15.3 Origin of the problem

The Charge-pump circuit shown in Figure II.15.1 consists of two capacitors and an n-channel MOS-transistor. The nodes gate, source, gate, and drain of the MOS-transistor are connected with the nodes 1, 2, 3, and Ground, respectively. In formulating the circuit equations, the transistor is replaced by four non-linear current sources in each of the connecting branches. They model the transistor.

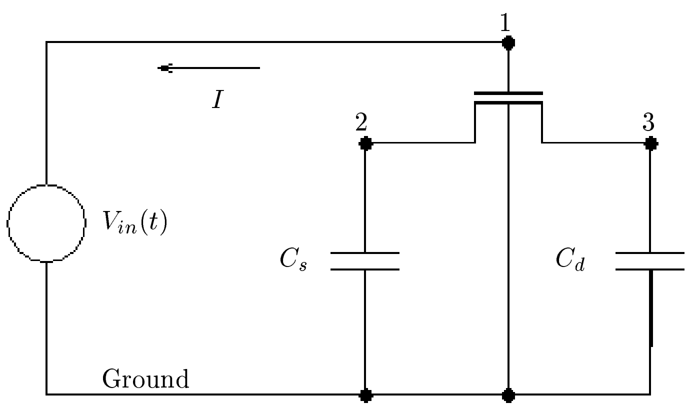

FIGURE II.15.1: *Circuit diagram of Charge-pump circuit (taken from [GDF95])*

After inserting the transistor model in the circuit, we get the final circuit, which can be obtained from the circuit in Figure II.15.1 by applying the following changes:

- Remove the transistor and replace it by a solid line between the nodes 2 and 3. The point where the lines 2–3 and 1–Ground cross each other becomes a node, which will be denoted by T.
- Add current sources between nodes 1 and T, between 2 and T and between 3 and T. There should also be a current source between the ground and node T, but as the node Ground does not enter the circuit equations, it will not be discussed. The currents produced by these sources are written as the derivatives of charges: current from 1 to T: Q_G', from T to 2: Q_S' and from T to 3: Q_D'. Here, the functions Q_G, Q_S and Q_D depend on the voltage drops U_1, $U_1 - U_2$ and $U_1 - U_3$, where U_i denotes the potential in node i.

The unknowns in the circuit are given by:

- The charges produced by the current sources: Y_{T1}, Y_{T2}, Y_{T3}. They are aliases for respectively Q_G, Q_S and Q_D. Consequently, Y'_{Ti} is the current between node T and node i.
- The charges Y_S and Y_D in the capacitors C_S and C_D.
- Potentials in nodes 1 to 3: U_1, U_2, U_3.
- The current through the voltage source $V_{in}(t)$: I.

In terms of these physical variables, the vector y introduced earlier reads

$$y = (Y_{T1}, Y_S, Y_{T2}, Y_D, Y_{T3}, U_1, U_2, U_3, I)^T.$$

Now, the following equations hold:

$$\begin{aligned} Y'_{T1} &= -I, \\ Y'_S + Y'_{T2} &= 0, \\ Y'_D + Y'_{T3} &= 0, \\ U_1 &= V_{in}(t). \end{aligned}$$

The charges depend on the potentials and are given by

$$\begin{aligned} Y_{T1} &= Q_G(U_1, U_1 - U_2, U_1 - U_3), \\ Y_S &= C_S \cdot U_2, \\ Y_{T2} &= Q_S(U_1, U_1 - U_2, U_1 - U_3), \\ Y_D &= C_D \cdot U_3, \\ Y_{T3} &= Q_D(U_1, U_1 - U_2, U_1 - U_3). \end{aligned}$$

The functions Q_G, Q_S and Q_D are given in the previous section.
Remark: the potential U_1 is known. Here, it is treated as an unknown in order to keep the formulation general and leaving open the possibility to extend the circuit. In addition, removing U_1 by hand contradicts a Computer Aided Design (CAD) approach in circuit simulation.

15.4 Numerical solution of the problem

The various components differ enormously in magnitude. Therefore, the absolute and relative input tolerances atol and rtol were chosen to be component-dependent. Furthermore, we neglect the index 2 variable y_9 in the error control of DASSL. This leads to the following input tolerances:

$$\begin{aligned} \text{atol}(i) &= \text{Tol} \cdot 10^{-6} && \text{for} \quad i = 1, \ldots, 5, \\ \text{atol}(i) &= \text{Tol} && \text{for} \quad i = 6, \ldots, 8, \\ \text{rtol}(i) &= \text{Tol} && \text{for} \quad i = 1, \ldots, 8, \\ \text{atol}(9) = \text{rtol}(9) &= 1000 && \text{for} \quad \text{DASSL}, \\ \text{atol}(9) = \text{rtol}(9) &= \text{Tol} && \text{for} \quad \text{other solvers}. \end{aligned}$$

The reference solution was computed using quadruple precision GAMD on an Alphaserver DS20E, with a 667 MHz EV67 processor, atol = rtol = 10^{-18}, $h_0 = 10^{-37}$.

Table II.15.1 and Figures II.15.3–II.15.4 present the run characteristics and the work-precision diagram, respectively. For the computation of the number of significant correct digits (scd), only the first component is taken into account. The second up to eighth component are ignored because these components are zero in the true solution; the ninth component is neglected because it was excluded

TABLE II.15.1: *Run characteristics.*

solver	Tol	mescd	scd	steps	accept	#f	#Jac	#LU	CPU
BIMD	10^{-5}	7.34	16.00	711	454	8827	454	711	0.0478
	10^{-7}	8.65	16.00	1125	688	15367	688	1125	0.0820
DDASSL	10^{-1}	0.93	0.14	447	438	604	369		0.0088
	10^{-3}	5.42	16.00	983	833	1659	853		0.0215
	10^{-5}	6.71	3.43	1737	1487	2903	1309		0.0361
	10^{-7}	6.09	3.32	3059	2587	4945	2058		0.0595
GAMD	10^{-1}	2.11	1.51	320	200	3735	200	320	0.0166
	10^{-3}	2.85	2.69	350	220	4786	220	350	0.0205
	10^{-5}	4.78	5.12	620	370	14890	320	570	0.0547
	10^{-7}	4.94	4.75	870	510	22340	410	770	0.0791
PSIDE-1	10^{-1}	1.17	0.37	938	839	9843	140	3752	0.0742
	10^{-5}	2.64	4.47	1366	1068	13424	160	5424	0.1005
	10^{-7}	9.05	16.00	2425	1555	24331	300	9616	0.1835

from DASSL's error control. For the mescd we consider all the components. The first component of the reference solution equals $0.1262800429876759 \cdot 10^{-12}$ at the end of the integration interval. We remark that the magnitude of this component is at most 10^{-10}. For the work-precision diagram, we used: Tol $= 10^{-(1+m/2)}$, $m = 0, 1, \ldots, 14$; h0 $= 10^{-6} \cdot$ Tol for BIMD, GAMD, MEBDFDAE, MEBDFI, RADAU and RADAU5. From Table II.15.1 and Figure II.15.3 we see that the numerical solution computed by DASSL results for some rather large values of Tol in an scd value of 15.4, which equals the accuracy of the reference solution.

Figure II.15.2 shows the behavior of the solution over the integration interval. Only the last four components have been plotted, since they are the physically important quantities. The other five components refer to charge flows inside the transistor, which are quantities the user is not interested in. These components have a similar behavior as the components 6, 7 and 8, but their magnitude is at most 10^{-10}.

The failed runs are in Table II.15.2; listed are the name of the solver that failed, for which values of m this happened, and the reason for failing.

References

[GDF95] M. Günther, G. Denk, and U. Feldmann. How models for MOS transistors reflect charge distribution effects. Technical Report 1745, Technische Hochschule Darmstadt, Fachbereich Mathematik, Darmstadt, 1995.

[MM08] F. Mazzia and C. Magherini. *Test Set for Initial Value Problem Solvers, release 2.4*. Department of Mathematics, University of Bari and INdAM, Research Unit of Bari, February 2008. Available at http://www.dm.uniba.it/~testset.

TABLE II.15.2: *Failed runs.*

solver	m	reason
BIMD	0	floating invalid
BIMD	4	too many consecutive Newton failures
BIMD	$3, 5, 7$	floating divide by zero
DASSL	2	error test failed repeatedly
DASSL	$4, 7$	floating overflow
DASSL	14	corrector failed to converge repeatedly
MEBDFDAE	$0, 1, \ldots, 14$	stepsize too small
MEBDFI	$0, 1, \ldots, 10$	floating invalid
MEBDFI	$11, 12, 13, 14$	stepsize too small
PSIDE-1	$4, 13, 14$	stepsize too small
RADAU	$0, 1, \ldots, 14$	stepsize too small
RADAU5	$0, 1, \ldots, 10$	floating invalid
RADAU5	$11, \ldots, 14$	stepsize too small

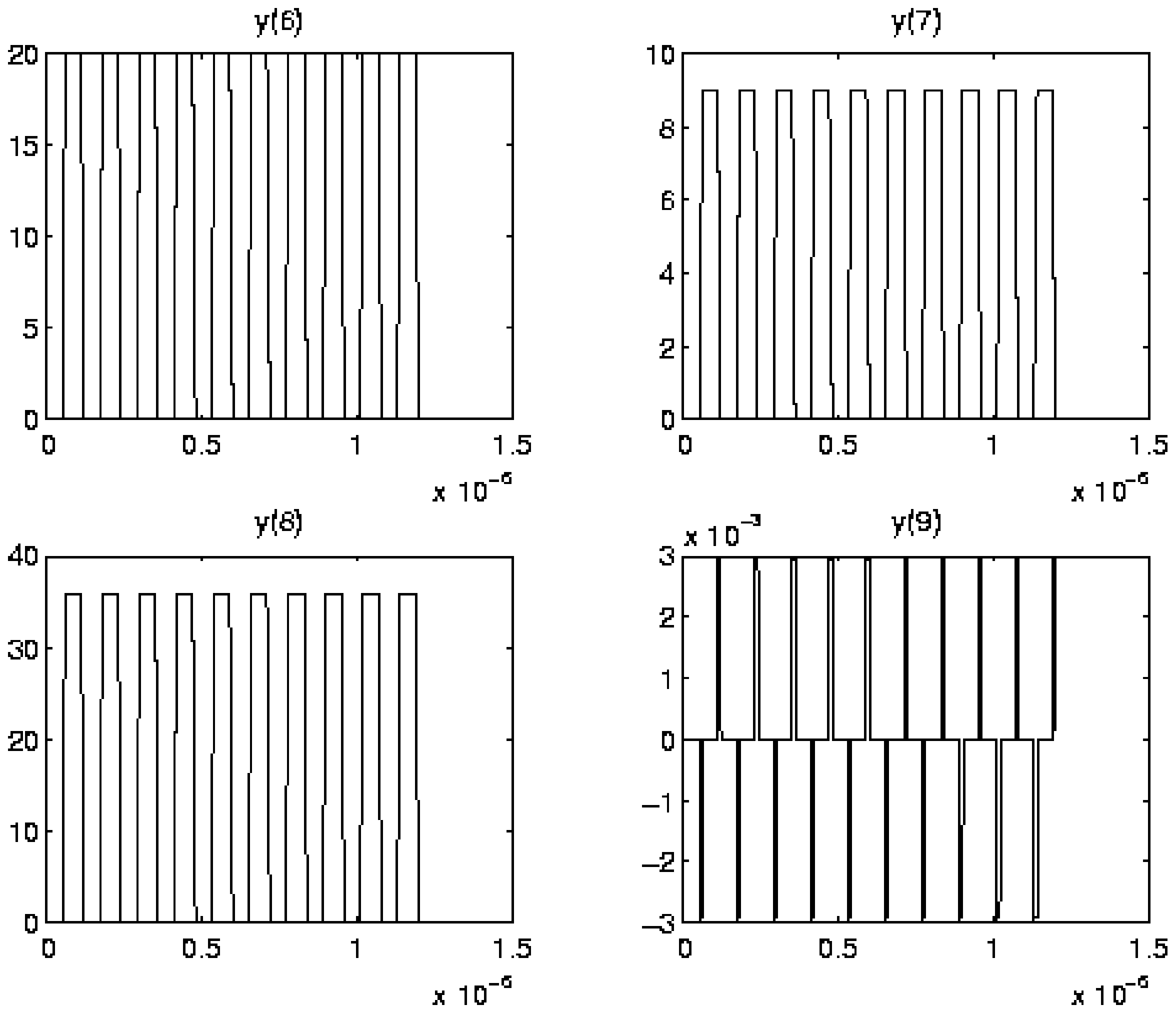

FIGURE II.15.2: *Behavior of the solution over the integration interval.*

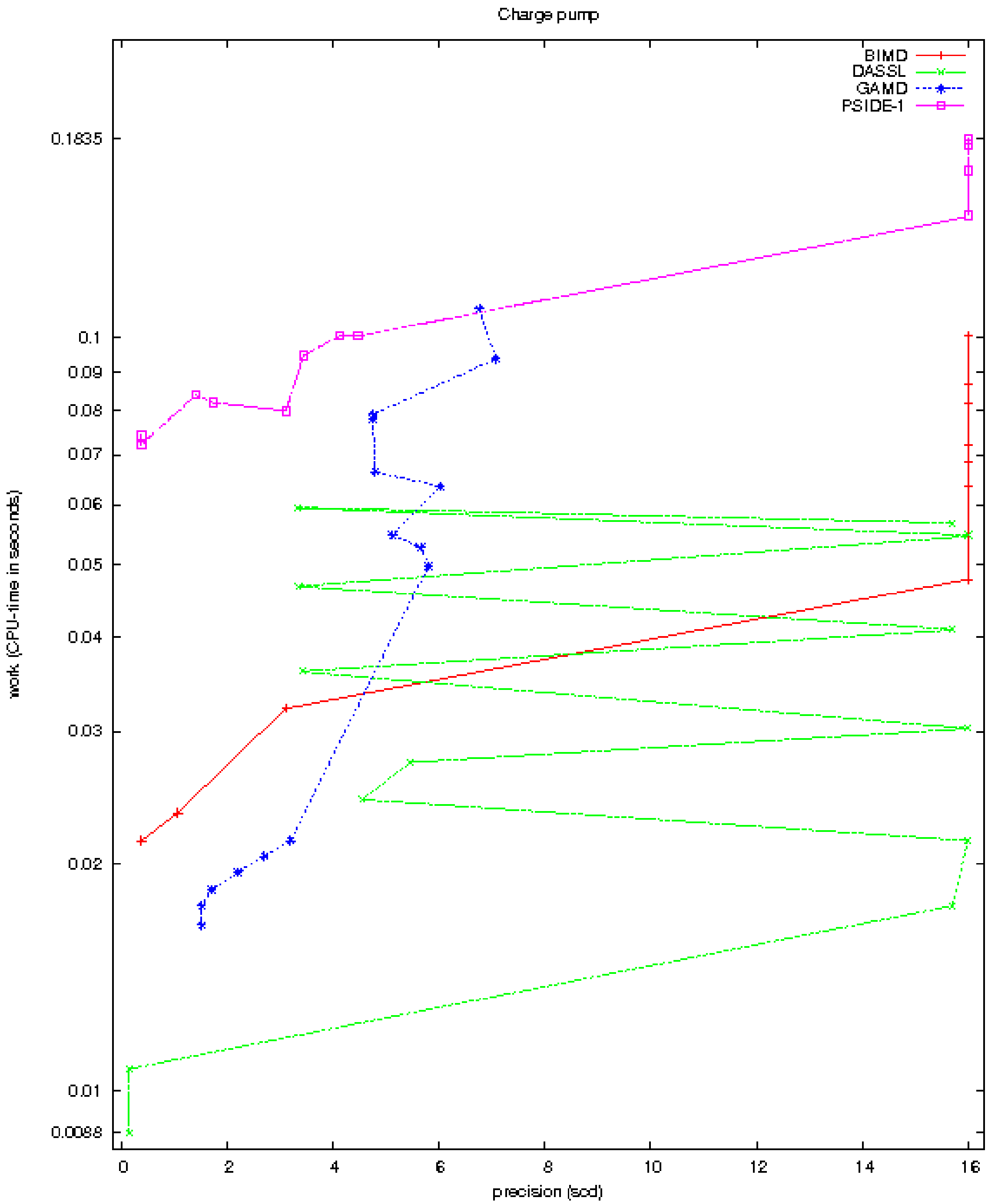

FIGURE II.15.3: *Work-precision diagram (scd versus CPU-time).*

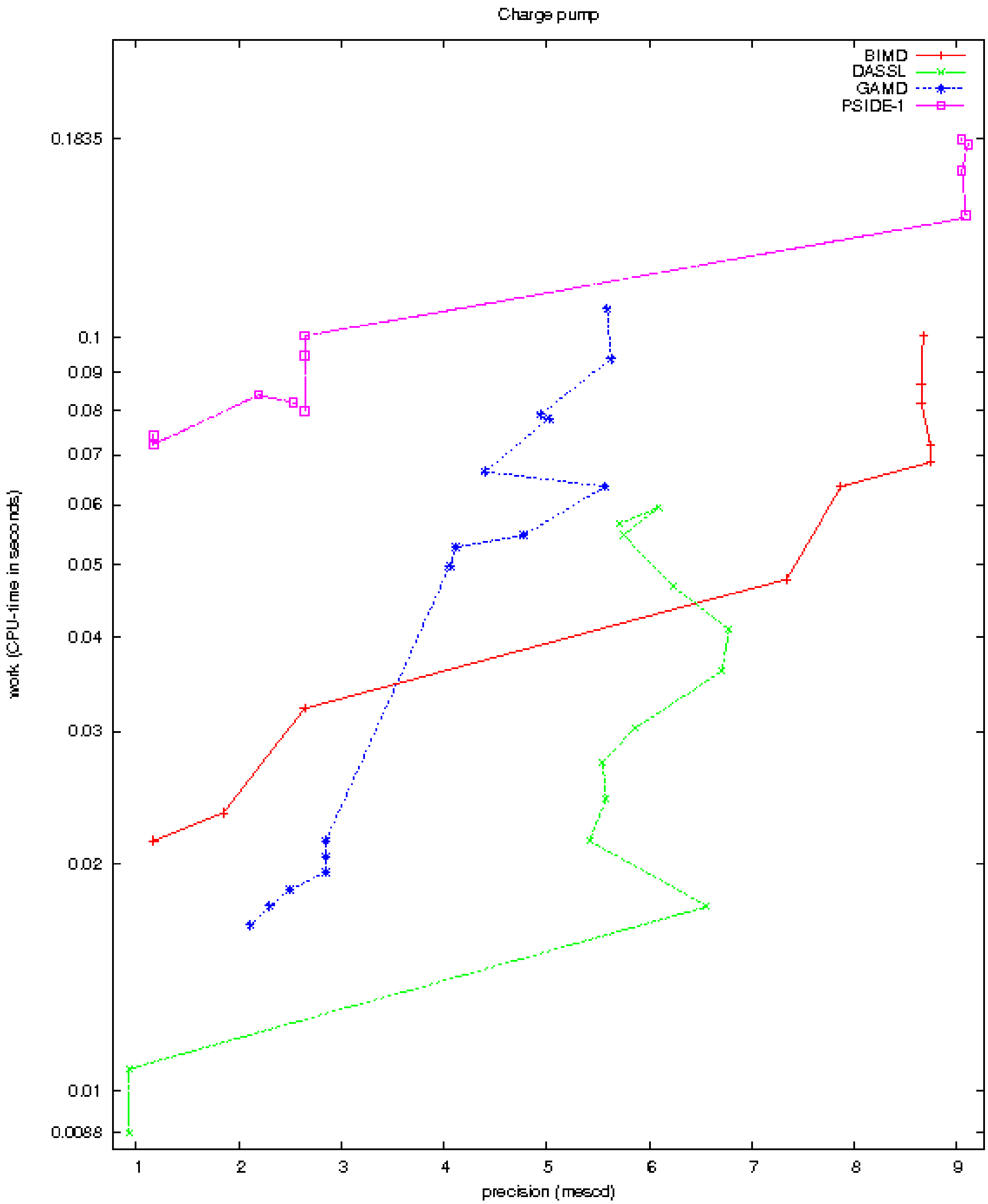

FIGURE II.15.4: *Work-precision diagram (mescd versus CPU-time).*