An introduction to the Virtual Element Method with a focus on the Navier-Stokes equation

Teacher
Giuseppe Vacca
https://sites.google.com/unimib.it/vaccagiuseppe/home

Course Website
https://sites.google.com/unimib.it/vaccagiuseppe/teaching

Course description
The Virtual Element Method (VEM) is a technology introduced in 2013 by Beirão da Veiga, Brezzi, Cangiani, Manzini, Marini, Russo for the discretization of partial differential equations.

The VEM can be interpreted as a novel approach that generalizes the classical Finite Element Method to arbitrary even non-convex element-geometry.
By avoiding the explicit integration of the shape functions that span the discrete Galerkin space and introducing a novel construction of the associated stiffness matrix, the VEM acquires very interesting properties and advantages with respect to more standard Galerkin methods yet keeping the same coding complexity.

The course will present, at the same time, the fundamental theoretical background of Virtual Elements, as well as their use for fluid-dynamic problems, and the basic tools and tricks for their actual implementation (together with practical coding sessions).

Course period
October-November 2023

SSD
MAT/08

Course References
[1] L. Beir˜ao da Veiga, F. Brezzi, A. Cangiani, G. Manzini, L.D. Marini, A. Russo, Basic principles of virtual element methods, Mathematical Models and Methods in Applied Sciences, 23(1):199-214 (2013).
[2] L. Beir˜ao da Veiga, F. Brezzi, L.D. Marini, A. Russo, the hitchhiker's guide to the virtual element methods, Mathematical Models and Methods in Applied Sciences, 24(8):1541-1573 (2014).
[3] L. Beir˜ao da Veiga, C. Lovadina, G. Vacca, Divergence free Virtual Elements for the Stokes problem on polygonal meshes, Mathematical Modelling and Numerical Analysis, 51(2):509-535 (2017).
[4] L. Beir˜ao da Veiga, C. Lovadina, G. Vacca, Virtual Elements for the Navier-Stokes problem on polygonal meshes, SIAM Journal on Numerical Analysis, 56(3):1210-1242 (2018).

Credits and Hours

2 credits, 16 hours lectures.

Exam Modality

Two alternatives are available to the student to pass this exam:

1) Paper presentation. Students present the content of some papers suggested by the teacher.
2) Project. Students implement and experimentally validate an algorithm or its variation from a paper suggested by the teacher. Projects can be done in groups of students.

Teacher CV

Curriculum

Teacher Main Publications

[1] G. Vacca, L. Beir˜ao da Veiga, Virtual Element Methods for parabolic problems on polygonal meshes, Numerical Methods for Partial Differential Equations, 31(6):2110-2134 (2015).
[2] F. Gardini, G. Vacca, Virtual Element Method for Second Order Elliptic Eigenvalue Problems, IMA Journal of Numerical Analysis, 38(4):2026-2054 (2018).
[3] G. Vacca, An H1-conforming virtual element for Darcy and Brinkman equations, Mathematical Models and Methods in Applied Sciences, 28(1):159-194 (2018).
[4] L. Beir˜ao da Veiga, C. Lovadina, G. Vacca, Virtual Elements for the Navier-Stokes problem on polygonal meshes, SIAM Journal on Numerical Analysis, 56(3):1210-1242 (2018).
[5] L. Beir˜ao da Veiga, A. Russo, G. Vacca, The Virtual Element Method with curved edges, Mathematical Modelling and Numerical Analysis, 53(2):375-404 (2019).
[6] L. Beir˜ao da Veiga, C. Canuto, R. H. Nochetto, G. Vacca, Equilibrium analysis of an immersed rigid leaflet by the virtual element method, Mathematical Models and Methods in Applied Sciences, 31(7):1323-1372 (2021).
[7] L. Beir˜ao da Veiga, F. Dassi, G. Vacca, Vorticity-stabilized Virtual Elements for the Oseen Equation, Mathematical Models and Methods in Applied Sciences, 31(14):3009-3052 (2021).
[8] L. Beir˜ao da Veiga, G. Vacca, Sharper error estimates for Virtual Elements and a bubble-enriched version, SIAM Journal on Numerical Analysis, 60(4):1853-1878 (2022).
[9] P. F. Antonietti, G. Vacca, M. Verani Virtual Element Method for the Navier–Stokes Equation coupled with the Heat Equation. To appear in IMA Journal of Numerical Analysis (2022).
[10] L. Beir˜ao da Veiga, C. Canuto, R. H. Nochetto, G. Vacca, M. Verani, Adaptive VEM: Stabilization-Free A Posteriori Error Analysis. SIAM Journal on Numerical Analysis, 61(2):399-494 (2023).