# Geodesics in Spacetime Manifolds: a Variational Approach

# Teacher

Anna Maria Candela (https://www.dm.uniba.it/it/members/candela)

#### **Course MS Teams**

Code: hscbge7

#### **Course description**

Exact solutions of the Einstein's field equations such as the classical Gödel Universe or also Exact Gravitational Waves are particular models of Lorentzian manifolds, while trajectories of free falling particles or of light rays can be described as geodesics in the chosen spacetime. Hence, investigating the geometric properties of a spacetime are interesting from both the physical and the mathematical viewpoint.

So, the aim of this course is, firstly, introducing the basic topics on semi-Riemannian Geometry pointing out the main differences between Riemannian and Lorentzian manifolds. Then, by means of suitable variational principles and by using some abstract existence theorems for critical points of C<sup>1</sup> functionals in Banach spaces, we look for geodesics in some "good" spacetimes. In particular, we study the geodesic connectedness of some families of Lorentzian manifolds and outline some existence results for timelike and lightlike geodesics in stationary spacetimes. Moreover, we highlight some results on Göde-type spacetimes and on Gravitational Waves.

#### Course period

January-February 2024

# SSD MAT/05

#### **Course References**

- [1] A.M. Candela and M. Sánchez, Geodesics in semi-Riemannian Manifolds: Geometric Properties and Variational Tools, In: "Recent developments in pseudo-Riemannian Geometry" (D.V. Alekseevsky & H. Baum Eds), Special Volume in the ESI-Series on Mathematics and Physics, EMS Publishing House, 2008, pp. 359-418 http://arxiv.org/PS\ cache/math/pdf/0610/0610144v2.pdf
- [2] A.M. Candela, J.L. Flores and M. Sánchez, On General Plane Fronted Waves. Geodesics, Gen. Relativity Gravitation 35 (2003), pp. 631-649.
- [3] B. O'Neill, Semi-Riemannian Geometry with Applications to Relativity, Academic Press Inc., New York, 1983
- [4] M. Struwe, Variational Methods. Applications to Nonlinear Partial Differential Equations and Hamiltonian Systems, 4th Ed., Ergeb. Math. Grenzgeb. (3) 34, Springer-Verlag, Berlin, 2008
- [5] Some Notes

#### **Credits and Hours**

3 CFU of lectures for a total of 24 hours.

# **Exam Modality**

Paper presentation: students present the content of one paper suggested by the teacher. No groups are allowed.

#### **Teacher CV**

See the attached file

# **Teacher Main Publications**

- [1] A.M. Candela, K. Perera and C. Sportelli, On a class of supercritical N-Laplacian problems, *Nonlinear Anal. Real World Appl.* **71** (2023), Article 103817. DOI:10.1016/j.nonrwa.2022.103817
- [2] A.M. Candela, A. Salvatore and C. Sportelli, Bounded solutions for quasilinear modified Schrödinger equations, *Calc. Var. Partial Differential Equations* 61 (6) (2022), Article 220. DOI:10.1007/s00526-022-02328-y
- [3] A.M. Candela and A. Salvatore, Existence of radial bounded solutions for some quasilinear elliptic equations in IR<sup>N</sup>, *Nonlinear Analysis* **191** (2020), Article 111625 (26 pages). DOI:10.1016/j.na.2019.111625
- [4] A.M. Candela, G. Palmieri and A. Salvatore, Multiple solutions for some symmetric supercritical problems, *Commun. Contemp. Math.* 22 (2020), Article 1950075 (20 pages). DOI:10.1142/S0219199719500755
- [5] A.M. Candela and G. Palmieri, A multiplicity result for a generalized p-Laplacian type problem with asymptotically p-linear terms, *Calc. Var. Partial Differential Equations* (2017), 56:72 (39 pages). DOI:10.1007/s00526-017-1170-4
- [6] R. Bartolo, A.M. Candela and J.L. Flores, Connectivity by geodesics on globally hyperbolic spacetimes with a lightlike Killing vector field, *Rev. Mat. Iberoam.* 33 (2017), pp. 1-28. DOI:10.4171/rmi/926
- [7] R. Bartolo, A.M. Candela and A. Salvatore, p-Laplacian problems with nonlinearities interacting with the spectrum, *NoDEA Nonlinear Differential Equations Appl.* **20** (2013), pp. 1701-1721. *DOI:10.1007/s00030-013-0226-1*
- [8] A.M. Candela, A. Romero and M. Sánchez, Completeness of the trajectories of particles coupled to a general force field, *Arch. Ration. Mech. Anal.* 208 (2013), pp. 255-274. DOI:10.1007/s00205-012-0596-2
- [9] A.M. Candela and G. Palmieri, Infinitely many solutions of some nonlinear variational equations, Calc. Var. Partial Differential Equations 34 (2009), pp. 495-530. DOI:10.1007/s00526-008-0193-2
- [10] A.M. Candela, J.L. Flores and M. Sánchez, Global hyperbolicity and Palais-Smale condition for action functionals in stationary spacetimes, *Adv. Math.* 218 (2008), pp. 515-536. DOI:10.1016/j.aim.2008.01.004

# CURRICULUM VITÆ OF ANNA MARIA CANDELA

Personal Information	<ul> <li>Dipartimento di Matematica Università degli Studi di Bari Aldo Moro via E. Orabona, 4 70125 Bari (Italy)</li> </ul>
	<ul> <li><u>annamaria.candela@uniba.it</u></li> <li><u>https://www.dm.uniba.it/members/candela</u></li> </ul>
	Nationality Italian
Current Position:	
21.10.2022 - present	Member of the Academic Board <i>(Senato Accademico)</i> Università degli Studi di Bari Aldo Moro (Bari, Italy)
3.10.2022 - present	Head of the Department of Mathematics Università degli Studi di Bari Aldo Moro (Bari, Italy)
3.12.2018 - present	Full Protessor Dinartimento di Matematica
	Università degli Studi di Bari Aldo Moro (Bari, Italy)
	Sector: Mathematical Analysis
WORK EXPERIENCE	
October - December 2021	<b>Coordinator</b> of the GEP Team for writing the <i>Gender Equality Plan 2022-2024</i> of <i>Università degli Studi di Bari Aldo Moro</i> . The <i>Gender Equality Plan 2022-2024</i> was signed by the Rector on 23.12.2021.
May - December 2021	<b>Member</b> , and <b>Coordinator</b> on behalf of the Rector, of the working group on <i>Bilancio di</i> <i>Genere 2021</i> (Gender Report 2021) of <i>Università degli Studi di Bari Aldo Moro</i> . The <i>Bilancio di Genere 2021</i> was signed by the Rector on 23.12.2021.
1.10.2019 - 12.9.2022	<b>Vice Rector (Pro Rettore Vicario)</b> Università degli Studi di Bari Aldo Moro (Bari, Italy)
15.7.2004 - 2.12.2018	Associate Professor Dipartimento di Matematica Università degli Studi di Bari Aldo Moro (Bari, Italy) • Sector: Mathematical Analysis
5.8.1992 - 14.7.2004	Assistant Professor Facoltà di Scienze MM FF NN Università degli Studi di Bari Aldo Moro (Bari, Italy) • Sector: Mathematical Analysis
EDUCATION, TRAINING AND	
ABILITATION	
30.12.2013	Italian National Scientific Abilitation as Full Professor in Mathematical Analysis
1.11.1990 - 31.12.1994	Ph.D. Degree in Mathematics
	Università degli Studi di Pisa (Pisa, Italy)
	<ul> <li>Dissertation Title: "Molteplicità di soluzioni positive di problemi ellittici non lineari proprietà topologiche del dominio" (25.1.1996)</li> </ul>

1.5.1990 - 31.10.1990	Postgraduate scholarship of the <i>Consiglio Nazionale delle Ricerche</i> Scuola Normale Superiore (Pisa, Italy)
a.y. 1985/86 - a.y. 1988/89	Degree in Mathematics Università degli Studi di Bari Aldo Moro (Bari, Italy)
	<ul> <li>Dissertation Title: "Soluzioni periodiche di assegnata energia di sistemi Hamiltoniani" (20.11.1989)</li> </ul>
s.y. 1980/81 - s.y. 1984/85	High school diploma at <i>Liceo Scientifico</i> (scientific oriented high school) Liceo Scientifico in Putignano (Bari, Italy)
PERSONAL SKILLS	
Mother tongue	Italian
Other languages	English (proficient user) French (basic user) Spanish (basic user)
Job-related skills	<ul> <li>Organizational skills</li> <li>Relational skills and ability to collaborate with colleagues</li> <li>Willingness to learn</li> <li>Communication skills</li> <li>Attention to detail</li> </ul>
Digital skills	<ul> <li>Microsoft Office 365</li> <li>Latex</li> <li>Wolfram Mathematica</li> <li>Canva</li> </ul>
PROJECTS (active)	
20.8.2019 - 19.8.2023	<b>Local coordinator</b> of the granted national research MIUR-PRIN project 2017JPCAPN_05 "Ovalitative and quantitative aspects of nonlinear PDFs" (PL B. Sciunzi)
27.9.2022 - 14.10.2023	co-investigator of the granted UniBa project "STEPS: STEerability and controllability of PDES in Agricultural and Physical models". Horizon Europe Seeds S51 (P.I. A. Salvatore)
1.9.2022 - 31.8.2025	<b>co-investigator</b> of the granted national research MUR-NRRP project Mission 4 Component 2 Investment 1.4 " <i>National Centre on HPC, Big Data and Quantum Computing</i> ", Spoke 10 " <i>Quantum Computing</i> ", funded by the European Union – NextGenerationEU, project code MUR CN00000013, CUP: H93C22000450007 (P.I. R. Bellotti)
1.9.2021 - 31.8.2025	<b>co-investigator</b> of the granted Spanish project <i>"Geometría semi-Riemanniana y flujos geométricos en Física-Matemática"</i> , PID2020-116126GB-I00, Ministerio de Ciencia e Innovación (P.I. M. Sánchez Caja and F. Martín Serrano)
EDITORIAL ACTIVITY	
2013 - present	Assistant Managing Editor of "Mediterranean Journal of Mathematics" (Birkhäuser, Editor-in-Chief Francesco Altomare)
2010 – present	Member of the Editor Board of " <i>Mediterranean Journal of Mathematics</i> " (Birkhäuser, Editor-in-Chief Francesco Altomare)

2014 - 2022	Associate Editor of "Boundary Value Problems" (Springer, Editor-in-Chief Kanishka Perera, Vicentiu D. Badulescu, Xianhua Tang)
2022 – present	<b>Guest Editor</b> of the Special Issue on "Partial Differential Equations and Semigroups in Applied Analysis" (A.M. Candela, G. Fragnelli, G.R. Goldstein, S. Lucente, A. Rhandi & S. Romanelli Eds). Discrete Contin. Dyn. Syst. Ser. S (to appear).
2022 – 2023	<b>Co-editor</b> of the book <i>"Recent Advances in Mathematical Analysis". Celebrating the</i> <i>70th Anniversary of Francesco Altomare</i> (A.M. Candela, M. Cappelletti Montano & E. Mangino Eds). <i>Trends Math.</i> Birkhäuser, 2023
2021 - 2022	<b>Guest Editor</b> of the Special Issue on "Analysis Motivated by the Applications. In memory of Rosa Maria Mininni" (A.M. Candela, G.R. Goldstein, J. Goldstein & S. Romanelli Eds), Discrete Contin, Dyn. Syst. Ser. S <b>15</b> (12) (2022)
2012 - 2013	<b>Guest Editor</b> of the Special Issue on "Evolution Equations and Mathematical Models in the Applied Sciences" (S. Romanelli, A.M. Candela, R.M. Mininni, A. Pugliese & M. De Giosa Eds), Discrete Contin, Dyn. Syst. Ser. S <b>6</b> (3) (2013).
2012 - 2013	<b>Co-editor</b> of the book <i>"Il Castello Aragonese di Taranto in 3D nell'evoluzione del paesaggio naturale"</i> (G. Mastronuzzi, L. Boccardi, A.M. Candela, C. Colella, G. Curci, F. Giletti, M. Milella, C. Pignatelli, A. Piscitelli, F. Ricci & P. Sansò Eds), DIGILABS s.a.s., Bari, 2013
PhD ADVISOR	
1.10.2019 - 31.12.2022	Caterina Sportelli (XXXV Cycle, PhD Research Program in Computer Science and Mathematics, Università degli Studi di Bari Aldo Moro) Dissertation Title: <i>"Solutions of Nonlinear PDEs: Variational and Topological Approaches"</i>
November 2005 - April 2007	(21.3.2023) Valeria Luisi (XIX Cycle, PhD Research Program in Mathematics, Università degli Studi di Bari Aldo Moro) Dissertation Title: "Some results about Fermat Principle and geodesic completeness in Lorentzian manifolds" (18.5.2007)
TEACHING	
a.y. 2013/2014 - present	<i>Analisi Superiore 2</i> (Advanced Mathematical Analysis), second level degree in Mathematics
a.y. 2021/2022 - present	Analisi Matematica n.3 (Mathematical Analysis no. 3), first level degree in Mathematics,
a.y. 2021/2022 - present a.y. 2020/21	<i>Analisi Matematica n.4</i> (Mathematical Analysis no. 4), first level degree in Mathematics, <i>Finsler Geometry and Spacetimes</i> (in cooperation with Miguel Angel Javaloyes Victoria), School of the Ph.D. in Computer Science and Mathematics
INSTITUTIONAL & ACADEM	IC
2013 - present	ES Member of the Boarding School of the Ph.D. in Computer Science and Mathematics
	(UniBa)
22.12.2020 – present	Scientific Director of the Framework Agreement for the Scientific Cooperation between Università degli Studi di Bari Aldo Moro and Fondazione E. Amaldi
SCIENTIFIC ACTIVITY	
Papers	<b>86 research papers</b> already published or in press on international journals or books (databases <i>MathSciNet</i> <b>MR Author ID 321613,</b> Web of Science <b>Researcher ID I-6545-2012</b> , <i>Scopus</i> <b>Author ID 7005848392</b> ) <b>35 invited talks</b> at conferences both in Italy and abroad <b>18 invited talks</b> at universities or
TUIKS	et inner and at contenences south in tary and abroad, to invited taris at anversities of

	research centers both in Italy and abroad, <b>17 short communications</b> both at national and international workshops
Visiting (short period)	Universidad de Granada (Granada, Spain), International Erwin Schrödinger Institute for Mathematical Physics (Wien, Austria), Peking University (Beijing, China), Chern Institute of Mathematics (Nankai University, Tianjin, China), Università degli Studi del Salento (Lecce, Italy), Università degli Studi di Perugia (Perugia, Italy), Humboldt-Universität zu Berlin (Berlin, Germany), Universidad de Málaga (Málaga, Spain), Universidad de Sevilla (Sevilla, Spain), Università degli Studi di Milano - Bicocca (Milano, Italy), Università degli Studi di Roma "Tor Vergata" (Roma, Italy), Università degli Studi della Basilicata (Potenza, Italy), Università degli Studi di Urbino Carlo Bo (Urbino, Italy)
Workshop Committee Research field	Member of the Scientific and/or the Organizing Committee of 31 workshops Variational and Topological Methods applied to the study of Nonlinear Differential Equations
Research Topics	Geodesics and trajectories in semi-Riemannian manifolds; elliptic equations in open bounded domains of IR <sup>N</sup> with either homogeneous or nonhomogeneous boundary conditions; elliptic equations in unbounded domains of IR <sup>N</sup> ; Schrödinger equation and its generalizations; quasilinear elliptic problems of p-Laplacian type both in bounded and unbounded domains; existence and multiplicity theorems for unbounded functionals in Banach space
GENDER EQUALITY	
7.10.2022 - present	<b>Contact person</b> , on behalf of <i>Università degli Studi di Bari Aldo Moro</i> together with Prof. Giuseppe Pirlo, for the management and implementation of the activities envisaged by the Memorandum of Understanding " <i>No Women, No Panel</i> " with <i>RAI-Radiotelevisione</i> <i>Italiana</i> and <i>Comune di Bari</i>
30.11.2021 - present	<b>Coordinator</b> , on behalf of <i>Università degli Studi di Bari Aldo Moro</i> , of the project <i>STEAMiamoci</i> , launched by <i>Assolombarda</i> , which aims to establish a synergistic network of national and international companies, universities, bodies and associations, engaged in projects to enhance female talents in STEM professions Some lectures on UniBa Gender Report and Gender Equality Plan
ADDITIONAL	
INFORMATION	Advisor of some degree thesis of students in <i>Mathematics</i> both of first level and of second level degree and of some first level degree thesis in <i>Science and Technical Nautical Management</i> . Member of the some Selection Boards for positions as Associate Professor or Researcher in Italian universities Intense activity in the popularization of Mathematics at different levels, mainly giving lectures for high school students, undergraduates in scientific faculties and older people attending universities of the third age Referee of some international journals in Mathematics
30.4.2021 - present	Socio Ordinario della Classe di Scienze Fisiche, Mediche, Naturali of Accademia Pugliese delle Scienze

Bari, 10.04.2023

Alu Chorlo Constel