

Variational tools in Nonlinear PDEs

Teachers

Pietro d'Avenia www.pdavenia.altervista.org

Alessio Pomponio www.alessiopomponio.altervista.org

Course description (min 150, max 300 words)

Many physical phenomena, such as matter field equations and electromagnetic theory, are described by nonlinear partial differential equations.

In this course we present some variational and topological methods for the study of such equations, with particular reference to local and non-local elliptic PDEs.

Starting from the simplest nonlinear elliptic PDEs, with a power like nonlinearity, we want to explore what happens when we consider more general nonlinearities. Here, previous and simpler methods do not apply and different strategies are in order to overcome the technical difficulties.

A particular attention will be given to the behavior of the Palais Smale sequences that is fundamental to obtain existence and multiplicity results. One of the first difficulties is to prove the boundedness of the PS sequences. In this framework, we introduce the monotonicity trick, developed by Struwe and by Jeanjean, and the construction of special PS sequences that satisfy additional properties, like Pohozaev type identities.

Moreover, we show how to deal with the lack of compactness, due to unboundedness of the domain or to the presence of Sobolev critical exponents, to get finally existence and multiplicity results.

Course period

March-May 2024

SSD

MAT/05

Credits and Hours

2 credits of lectures for a total of 16 hours.

Exam Modality

Paper presentation. Students present the content of papers suggested by the teachers.

Teacher(s) CV

Attach or link a max 3 pages CV for each teacher proposing the course.

Teacher(s) Main Publications

List 10 main publications in the last 15 years for each teacher.

Pietro d'Avenia's Main Publications

1. A. Azzollini, P. d'Avenia, A. Pomponio, *On the Schrödinger-Maxwell equations under the effect of a general nonlinear term*, Ann. Inst. H. Poincaré Anal. Non Linéaire 27 (2010), no. 2, 779--791, doi: 10.1016/j.anihpc.2009.11.012.
2. P. d'Avenia, A. Pomponio, D. Ruiz, *Semiclassical states for the nonlinear Schrödinger equation on saddle points of the potential via variational methods*, J. Funct. Anal. 262 (2012), no. 10, 4600--4633, doi: 10.1016/j.jfa.2012.03.009.
3. P. d'Avenia, J. Mederski, *Positive ground states for a system of Schrödinger equations with critically growing nonlinearities*, Calc. Var. Partial Differential Equations 53 (2015), no. 3-4, 879--900, doi: 10.1007/s00526-014-0770-5.
4. P. d'Avenia, G. Siciliano, M. Squassina, *On fractional Choquard equations*, Math. Models Methods Appl. Sci. 25 (2015), 1447--1476, doi: 10.1142/S0218202515500384.
5. D. Bonheure, P. d'Avenia, A. Pomponio, *On the electrostatic Born-Infeld equation with extended charges*, Comm. Math. Phys. 346 (2016), no. 3, 877--906, doi:10.1007/s00220-016-2586-y.
6. P. d'Avenia, M. Squassina, *Ground states for fractional magnetic operators*, ESAIM Control Optim. Calc. Var. 24 (2018), no.1, 1--24, doi: 10.1051/cocv/2016071.
7. P. d'Avenia, G. Siciliano, *Nonlinear Schrödinger equation in the Bopp-Podolsky electrostatics: solutions in the electrostatic case*, J. Differential Equations 267 (2019), 1025--1065, doi: 10.1016/j.jde.2019.02.001.
8. D. Bonheure, P. d'Avenia, A. Pomponio, W. Reichel, *Equilibrium measures and equilibrium potentials in the Born-Infeld model*, J. Math. Pures Appl. 139 (2020), 35--62, doi: 10.1016/j.matpur.2020.05.001.
9. P. d'Avenia, J. Mederski, A. Pomponio, *Nonlinear scalar field equation with competing nonlocal terms*, Nonlinearity 34 (2021), 5687--5707, 10.1088/1361-6544/ac0d47.
10. P. d'Avenia, M.G. Ghimenti, *Multiple solutions and profile description for a nonlinear Schrödinger-Bopp-Podolsky-Proca system on a manifold*, Calc. Var. Partial Differential Equations 61 (2022), 223, doi: 10.1007/s00526-022-02341-1.

Alessio Pomponio's Main Publications

1. A. Azzollini, A. Pomponio, *On the Schrodinger equation in \mathbb{R}^N under the effect of a general nonlinear term*, Indiana Univ. Math. J., 58, (2009), 1361–1378, doi: 10.1512/iumj.2009.58.3576.
2. A. Azzollini, P. d'Avenia, A. Pomponio, *On the Schrodinger-Maxwell equations under the effect of a general nonlinear term*, Ann. Inst. H. Poincaré Anal. Non Linéaire, 27, (2010), 779–791, doi: 10.1016/j.anihpc.2009.11.012.
3. P. d'Avenia, A. Pomponio, D. Ruiz, *Semiclassical states for the Nonlinear Schrodinger Equation on saddle points of the potential via variational methods*, J. Funct. Anal., 262, (2012), 4600–4633, doi: 10.1016/j.jfa.2012.03.009.
4. A. Azzollini, P. d'Avenia, A. Pomponio, *Quasilinear elliptic equations in \mathbb{R}^N via variational methods and Orlicz-Sobolev embeddings*, Calc. Var. Partial Differential Equations, 49, (2014), 197–213, doi: 10.1007/s00526-012-0578-0.
5. A. Pomponio, D. Ruiz, *A Variational Analysis of a Gauged Nonlinear Schrodinger Equation*, J. Eur. Math. Soc., 17, (2015), 1463–1486, doi: 10.4171/JEMS/535.
6. A. Pomponio, D. Ruiz, *Boundary concentration of a Gauged Nonlinear Schrodinger Equation on large balls*, Calc. Var. Partial Differential Equations, 53, (2015), 289–316, doi: 10.1007/s00526-014-0749-2.
7. D. Bonheure, P. d'Avenia, A. Pomponio, *Electrostatic Born-Infeld equation with extended charges*, Communication in Mathematical Physics, 346, (2016), 877–906, doi: 10.1007/s00220-016-2586-y.
8. A. Pomponio, T. Watanabe, *Some quasilinear elliptic equations involving multiple p -Laplacians*, Indiana Univ. Math. J., 67, (2018), 2199–2224, doi: 10.1512/iumj.2018.67.7523.
9. D. Bonheure, P. d'Avenia, A. Pomponio, W. Reichel, *Equilibrium measures and equilibrium potentials in the Born-Infeld model*, J. Math. Pures Appl., 139, (2020), 35–62, doi: 10.1016/j.matpur.2020.05.001.
10. A. Azzollini, A. Pomponio, *Positive energy static solutions for the Chern-Simons-Schrodinger system under a large-distance fall-off requirement on the gauge potentials*, Calc. Var. Partial Differential Equations, 60, 165 (2021), doi: 10.1007/s00526-021-02031-4.

Pietro d'Avenia

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Personal details

- date of birth 28 January 1974
birth place Canosa di Puglia (BT)
address via Roma, 9
(home) 76013 Minervino Murge (BT), Italy
+39 0883 694556
address Politecnico di Bari
(office) Dipartimento di Meccanica, Matematica e Management
Via Orabona, 4
70125, Bari, Italy.
military service Absolved as Sottotenente di Complemento in Arma di Artiglieria at 52° Reggimento di Artiglieria Campale Semovente “Torino” (Vercelli) from 15 June 1999 to 15 August 2000.
present position Associate Professor of Mathematical Analysis at Politecnico di Bari.

Main steps in education and employment history

- 1993–1998 **Laurea in Matematica**, at *Università degli Studi di Bari*.
Score: 110 cum laude/110.
Thesis in Nonlinear Analysis “*Onde solitarie in tre dimensioni spaziali con vincolo topologico*” (supervision of Professor Donato Fortunato and Professor Lorenzo Pisani).
1998–1999 **CNR research fellow**, at *Università degli Studi di Bari*.
2000–2002 **Research fellow**, at *Università degli Studi di Bari*.
2006–2019 **Ricercatore in Mathematical Analysis**, at *Politecnico di Bari*.
2019–now **Associate Professor of Mathematical Analysis**, at *Politecnico di Bari*.

Publications

Author of more than 40 papers published in international journals.

Conference talks, communications, seminars and minicourses

- 09 April 2001 *Onde Solitarie Topologiche*, Workshop within the research project “Metodi variazionali e topologici nello studio di fenomeni non lineari”, San Mommè (invited).
28 May 2001 *Mancaanza di compattezza in problemi ellittici non lineari*, PhD Course “Metodi Variazionali”, Università degli Studi di Bari (invited).
14 June 2001 *Lo Splitting Lemma per l'esistenza di onde solitarie nel modello BFP*, PhD Course “Metodi Variazionali”, Università degli Studi di Bari (invited).
29 August 2001 *Infinitely Many Solitary Waves in Three Space Dimensions*, Equadiff 10 - Czechoslovak International Conference on Differential Equations and Their Applications.
25 September 2007 *Sul sistema Klein-Gordon-Maxwell con condizioni di Neumann*, XVIII Congresso U.M.I., Bari.
15 September 2008 *Symmetrizations and Pólya-Szegö inequality*, XXXIII Scuola Estiva di Fisica Matematica, Ravello.
17 May 2010 *Infinitely many positive solutions for a Schrödinger-Poisson system*, Spring School in Nonlinear Partial Differential Equations, Bruxelles.
16 November 2010 *Existence and multiplicity for a general nonlinear Schrödinger-Maxwell system*, Departamento de Análisis Matemático, Universidad de Granada (invited).
March–April 2012 Series of seminars (four) within the PhD course *Variational tools in nonlinear elliptic equations*, Università degli Studi della Basilicata (invited).

- 04 June 2012 *Generalized Schrödinger-Poisson type systems*, Spring School in Nonlinear Partial Differential Equations, Bruxelles.
- September 2013 Minicourse *Semilinear elliptic equations: existence results via a variational approach*, Instituto de Matemática e Estatística, Universidade de São Paulo (invited).
- 03 April 2014 *Positive ground states for some critical elliptic systems*, $P(n)$ Seminar, Sapienza Università di Roma (invited).
- 21 May 2014 *Ground states for a critical elliptic system*, Dipartimento di Matematica e Applicazioni, Università di Milano-Bicocca (invited).
- 07 October 2014 *Chern-Simons-Schrödinger equation with a general nonlinearity*, Wydział Matematyki i Informatyki, Uniwersytet Mikołaja Kopernika, Toruń (invited).
- 08 September 2015 *Born-Infeld equations in the electrostatic case: radially symmetric and bounded charge distributions*, Workshop in Nonlinear PDEs, Bruxelles (invited).
- 15 September 2015 *Born-Infeld equations in the electrostatic case: superposition of point charges*, VII Symposium on Nonlinear Analysis, Toruń (invited).
- 26 January 2016 *A multiplicity result for a general nonlinear Chern-Simons-Schrödinger equation*, Fakultät für Mathematik, Karlsruher Institut für Technologie (invited).
- 12 April 2016 *Ground states for fractional magnetic operators*, Wydział Matematyki i Informatyki, Uniwersytet Mikołaja Kopernika, Toruń (invited).
- 12 January 2017 *On the Born-Infeld electromagnetic theory: the electrostatic case*, Instytut Matematyczny, Polska Akademia Nauk, Warszawa (invited).
- 21 March 2017 *On the Born-Infeld electromagnetic theory: the electrostatic case*, Instituto de Matemática, Estatística e Computação Científica, Departamento de Matemática, Universidade Estadual de Campinas (invited).
- 24 March 2017 *On the Born-Infeld equation*, Instituto de Matemática e Estatística, Departamento de Matemática, Universidade de São Paulo (invited).
- 28 March 2017 *Ground states for fractional magnetic operators*, Universidade Federal do ABC, Santo André, São Paulo, (invited).
- 31 March 2017 *The electrostatic Born-Infeld equation*, Instituto de Ciências Exactas, Departamento de Matemática, Universidade de Brasília (invited).
- 23 May 2017 *Vortex ground states for Klein-Gordon-Maxwell-Proca type systems*, International Conference on Elliptic and Parabolic Problems, Gaeta (invited).
- 27 October 2017 *Nonlinear fractional magnetic Schrödinger equations*, Instituto de Ciências Exactas, Departamento de Matemática, Universidade de Brasília (invited).
- 12 December 2017 *On the Born-Infeld equation*, Intensive week of PDEs at Spa, Spa (invited).
- 19 February 2018 *On the Born-Infeld equation*, X Workshop de Verão em Matemática, Brasília (invited).
- 16 May 2018 *Vortex ground states for Klein-Gordon-Maxwell-Proca type systems*, Instituto de Ciências Exactas, Departamento de Matemática, Universidade de Brasília (invited).
- 13 September 2018 *Nonlinear Schrödinger-Bopp-Podolsky system*, Nonlinear Analysis and PDEs, Caserta (invited).
- 21 May 2019 *Nonlinear Schrödinger-Bopp-Podolsky system*, International Conference on Elliptic and Parabolic Problems, Gaeta (invited).
- 12 February 2021 *Ground states for an Hartree-Fock type system*, XIII Summer Workshop in Mathematics, Brasília (invited).
- 03 September 2021 *Ground states for an Hartree-Fock type system*, Nonlinear Elliptic PDEs in Ancona (invited).
- 09 September 2021 *Ground states for an Hartree-Fock type system*, 8th International Conference on Mathematics and Informatics, Târgu Mureș (invited).
- 21 September 2021 *Ground states for an Hartree-Fock type system*, Three days of PDEs, friendship, love and nonlinearities, Pisa (invited).
- 06 November 2021 *Ground states for an Hartree-Fock type system*, Tutti insieme... nonlinearmente, Manfredonia (invited).
- 17 January 2022 *On a mixed dispersion nonlinear Schrödinger equation*, XIV Summer Workshop in Mathematics Mat/UnB, Brasília (invited).
- 12 May 2022 *On a mixed dispersion nonlinear Schrödinger equation*, Sapienza Università di Roma ($P(n)$ Seminar, invited).

- 18 May 2022 *On a mixed dispersion nonlinear Schrödinger equation*, Dipartimento di Matematica, Università degli Studi di Pisa (invited).
- 14 July 2022 *On a mixed dispersion nonlinear Schrödinger equation*, Two nonlinear days in Urbino 2022, (invited).
- 26 July 2022 *Infinitely many entire solutions to a mixed dispersion NLS equation with generic nonlinearity: the positive mass case*, Topics in variational problems arising from models in physics, Będlewo (invited).
- 14 September 2022 *On a mixed dispersion nonlinear Schrödinger equation*, IV Workshop on Trends in Nonlinear Analysis, Cagliari (invited).

Editorial activity

- Editor for *Advances in Mathematical Physics*, *MAYFEB Journal of Mathematics*, *New Horizons in Mathematical Physics*, *International Journal of Theoretical and Applied Mathematics*, *Contemporary Mathematics*, *Symmetry*.
- Referee for several international journals
- Rewiever *Mathematical Reviews* from January 2007.
McGraw-Hill Italia, February 2009.

Research projects

- Principal Investigator *Onde solitarie*, FRA2011;
PDEs tra materia e campi elettromagnetici, FRA2016 (participants: Alessio Pomponio).
Qualitative and quantitative aspects of nonlinear PDEs, PRIN2017 2017JPCAPN, Scientific Responsible at Politecnico di Bari, National Principal Investigator Bernardino Sciunzi.
Equazioni e sistemi nonlineari di tipo ellittico, FRA 2019, (participants: Giuseppina Vannella).
- Participant to many national and international projects.

Scientific Events' Organization

- Intorno ad alcune applicazioni della Matematica*, Bari, November 18, 2009.
Critical Point Theory and Nonlinear Differential Problems, Canazei, 02–04 September 2015.
Seminars on Analysis and Geometry, Bari, February 22, 2017.
Session Variational problems and nonlinear PDEs, Italian-Polish Mathematical Meeting in Wrocław, 17–20 September 2018.
Session Variational problems and nonlinear PDEs, International Conference on Elliptic and Parabolic Problems, Gaeta, 20–24 May 2019.
Advances and Challenges in Nonlinear Analysis ... and Beyond! On the Occasion of Vieri Benci's 70th Birthday, Bari, 24–27 September 2019.

Schools and Workshops

Participant to several national and international events.

Grants

- May 1998–May 1999 Research fellow (n. 209.01.63) by Consiglio Nazionale delle Ricerche at Università degli Studi di Bari.
- Sept 2000–Aug 2002 Research fellow at Università degli Studi di Bari (assigned on 08 October 1999).
- 02 December 1999 Research fellow (n. 201.01.129) by Consiglio Nazionale delle Ricerche at Istituto per le Applicazioni del Calcolo “Mauro Picone” (non accepted: incompatible with Research Fellow at Università degli Studi di Bari).
- ASN 2018/2020, Q3 Habilitation as Professore Ordinario (Mathematical Analysis).
- 2014 - 2019 - 2020 Referee for a Research Fellow at Università degli Studi dell'Insubria.
Included in World's Top 2 % Scientists by Stanford University, Single Year 2019 and 2020

Bari, April 21, 2023

Pietro D'Amico

Curriculum vitae of Alessio Pomponio

PERSONAL DETAILS

Name: Alessio

Surname: Pomponio

Date of birth: 19/11/1976

Place of birth: Bari, Italy

Residence: Via Avvocato Michele Zella Milillo 11 70010 Casamassima (BA), Italy

Italian citizen

EDUCATION

2000. Graduated cum laude in Mathematics at the Università degli Studi di Bari, advisors D. Fortunato and L. Pisani.

2004. PhD at SISSA, Trieste, in Functional Analysis Sector, advisor A. Ambrosetti.

ACADEMIC POSITIONS

01/03/2005 - 29/02/2008 Assistant Professor – Mathematical Analysis – Polytechnic University of Bari.

01/03/2008 - 14/05/2015 Confirmed Assistant Professor – Mathematical Analysis – Polytechnic University of Bari.

30/12/2013 - 14/05/2015 License as Associate Professor – Mathematical Analysis (MAT/05).

15/05/2015 - 01/05/2022 Associate Professor – Mathematical Analysis – Polytechnic University of Bari.

28/03/2017 - 01/05/2022 License as Full Professor – Mathematical Analysis (MAT/05).

02/05/2022 - *present* Full Professor – Mathematical Analysis – Department of Mechanics, Mathematics and Management – Polytechnic University of Bari.

PRIZES

2016 James S.W. Wong JMAA Prize for the paper: A. Azzollini, A. Pomponio, "Ground state solutions for the nonlinear Schrödinger-Maxwell equations" J. Math. Anal. Appl. 345 (2008), no. 1, 90-108.

VISITING ACADEMIC POSITIONS

- March and June 2011, September 2012, April 2013, March 2019, visitor at Universidad de Granada, Spain.

- February 2013, May 2016, visitor at Université libre de Bruxelles, Belgium.

- July 2016, visitor at Karlsruher Institut für Technologie, Germany.

- September 2017, February and March 2019, March 2023, visitor at Instytut Matematyczny Polskiej Akademii Nauk, in Warsaw, Poland.

- February 2018, visitor at Nicolaus Copernicus University, in Torun, Poland.

- November 2022, visitor at New York University NYU and New Jersey Institute of Technology NJIT, United States of America.

MAIN TEACHING ACTIVITIES

- Instructor of many courses of Calculus, Calculus I and Calculus II at Polytechnic University of Bari since A.A. 2005/2006.

- Instructor of Advanced Mathematical Methods, (taught in English), at Polytechnic University of Bari, since A.A. 2020/2021.

PHD TEACHING

- Seminars cycle for the PhD course “Variational tools in nonlinear elliptic equations” at Università degli Studi della Basilicata, A.A. 2011/2012.

- Introduction to PDEs and Applications, Scuola di Dottorato del Polytechnic University of Bari, Cycle XXXVI, A.A. 2020/2021.

PARTECIPATIONS TO DOCTORAL COLLEGES

- Member of the doctoral college of Ph.D. in Mechanical Engineering and Management PH.D, cycle XXXVI, XXXVII and XXXVIII of Polytechnic University of Bari.

INSTITUTIONAL ASSIGNMENTS

2021 – present: delegate for internationalization at Department of Mechanics, Mathematics and Management, Polytechnic University of Bari.

2021 – present: member of the Research Committee of the Department of Mechanics, Mathematics and Management, Polytechnic University of Bari.

COMMITTEES

- Member of the Ph.D. Committee, Programa de doctorado de Física y Matematicas (FisyMat), Universidad de Granada, March 2017.

- Member of the Evaluation Committee for the admission to Ph.D. in Mechanical Engineering and Management, cycle XXXVI, of Polytechnic University of Bari, September-October 2020.

- Member of the Evaluation Committee for the final exam of Ph.D. in Mechanical Engineering and Management PH.D, cycle XXXIII, of Polytechnic University of Bari, April 2021.

- Member of the Evaluation Committee for the assignment of grants for collaboration in research activities of Polytechnic University of Bari, February 2022.

- Member of the Evaluation Committee for the admission to Double Degree in Management Engineering of Polytechnic University of Bari, April 2022.

- Member of the Evaluation Committee for a position of Associate Professor, in Mathematical Analysis at Department of Mechanics, Mathematics and Management of Polytechnic University of Bari, June-July 2022.

- Member of the Evaluation Committee for two positions of Researcher RTDB, in Mathematical Analysis at Department of Mathematics and Informatics of Università della Calabria, July-September 2022.

COORDINATOR OF FUNDED RESEARCH PROJECTS

- Fondo di Ricerca di Ateneo, Politecnico di Bari 2011.

- Progetto GNAMPA 2017, “Metodi matematici per lo studio di fenomeni fisici nonlineari”.

- Fondo di Ricerca di Ateneo, Politecnico di Bari 2021.

PARTICIPATION AT FUNDED RESEARCH PROJECTS

- Prin 2002, Prin 2004, coordinator A. Ambrosetti.

- Prin 2005, Prin 2007, Prin 2009, coordinator V. Benci.
- Azione Integrata Italia Spagna 2009, coordinators G. Cerami and M. Sanchez Caja.
- GNAMPA 2011, 2012, 2013, 2014, 2015, 2016, coordinator S. Cingolani.
- MIS - Mandat d'Impulsion Scientifique - F450814F (FNRS), coordinator D. Bonheure.
- National Science Centre, Poland (Grant Nos. 2013/09/B/ST1/01963 and 2014/15/D/ST1/03638), coordinator J. Mederski.
- Fondo di Ricerca di Ateneo, Politecnico di Bari 2016, coordinator P. d'Avenia.
- PRIN 2017JPCAPN "Qualitative and quantitative aspects of nonlinear PDEs", coordinator B. Sciunzi.
- Progetto GNAMPA 2022, coordinator A. Azzollini.
- Progetto GNAMPA 2023, coordinator T. Isernia.

ORGANIZATION

- Member of the Organizing Committee of the scientific meeting "Seminars on Analysis and Geometry", Polytechnic University of Bari, Bari, February 22, 2017.
- Member of the Scientific Committee and of the Organizing Committee of the DMMM Winter School "Mathematics for Engineering Applications", Polytechnic University of Bari, Bari, January 27–31, 2020.
- Member of the Organizing Committee of the scientific meeting "Qualitative and quantitative aspects of nonlinear PDEs" Bari, Polytechnic University of Bari and Università degli Studi di Bari Aldo Moro, September 05–09, 2022.
- Member of the Scientific Committee and of the Organizing Committee of the scientific meeting "Nonlinear Analysis in the Court of the General Relativity, on the occasion of Antonio Masiello's 60th birthday", Polytechnic University of Bari, Bari, February 09–10, 2023.

EDITORIAL ACTIVITY

- Associate editor of Journal of Mathematical Research and Applications, (2013-2015).
- Editorial Board Member of SCIREA Journal of Mathematics, (2018 - present).
- Editorial Board Member of Pure and Applied Mathematics Journal, (2018 - present).
- Editorial Board Member of Contemporary Mathematics, (2019 - present).
- Editorial Board Member of Mathematics, (2019 - present).

MEMBERSHIP OF SCIENTIFIC SOCIETIES

- Member of the Advisory Board of the Universal Scientific Education and Research Network (USERN).
- Member of Unione Matematica Italiana (U.M.I.).

Bari, 21 April 2023

